Inflation Dynamics and Exchange Rate Pass-Through in Nigeria: Evidence from Augmented Nonlinear New Keynesian Philips Curve

Usman A. Bello† and Aliyu R. Sanusi‡,§

This paper estimates a nonlinear augmented New Keynesian Philips Curve for Nigeria using the Smooth Transition Regression model for the period 1995Q1 to 2018Q2. The empirical evidence reveals the existence of two inflation regimes during the period under review. Food inflation, energy inflation, firms’ marginal cost, and imported inflation account for most of the changes in the prices of composite consumers’ basket in low exchange rate depreciation regime. However, the exchange rate solely explains price changes in the composite consumers’ basket when inflation switches to high regime. Similarly, the results show that regime change in inflation is largely caused by exchange rate (transition variable) depreciation or devaluation of the naira. Furthermore, the paper finds that the threshold in exchange rate devaluation (depreciation) that triggers a regime switch from low to high inflation regime is about N75 relative to a dollar. The speed of regime switch was found to be significantly high at about 70% per quarter. The paper argues that achieving exchange rate stability is a necessary condition for disinflation during this regime. Therefore, this paper recommends that monetary policy response to low inflation regime must target the various components of the consumption basket while effort to curtail persistent high inflation must include a stable exchange rate of the naira.

Keywords: Inflation, Regime switch, Philips curve, Threshold, Exchange rate.

JEL Classification: E12, E31, E52, E58

DOI: 10.33429/Cjas.10219.4/6

1.0 Introduction

Observed changes in the composite consumer price index (CPI, or the measure of headline of inflation) are often because of changes in prices of one or a few of the sub-components of the composite consumers’ basket.

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Indeed, most observed episodes of changes in the aggregate inflationary pressure are because of some complex dynamics within the composite index. For instance, while prices of some of the sub-components of the aggregate consumption basket may be rising, that of other sub-components may be falling. In such a case, therefore, the direction of the change in the composite consumer price index depends on the relative magnitude of the changes as well as the relative weights of the affected sub-components. Therefore, this means inflationary pressure observed in the consumers index may not necessarily be because of a general and one-directional increase in all its components. These possible asymmetries within the composite consumers’ basket may often be an important consideration for monetary policy response, to achieve effective disinflation. Indeed, monetary policy efforts for achieving disinflation could be undermined when central bank ignores these internal dynamics and targets aggregate, instead of the specific sub-component that is the sources of the observed changes. In addition, there may be important welfare implications for households or firms when monetary policy, say, tightens in response to changes in the aggregate index, when source of the change may indeed be single or few sub-components. This welfare implication arises because economic agents are non-representative and are heterogeneous in the different items and weights that makes up their actual (individual) consumption baskets. Thus, there is need to pay greater attention to the dynamics that occurs within the composite consumer price index, and to disaggregate it to isolate the most unstable components for specific targeting.

The internal dynamics of the composite measure of inflation may also be important to monetary policy to effectively anchor inflation expectations. For instance, by responding to the specific sub-components of the price indices, which economic agents are better able to relate to, the monetary policy authority may raise their belief about its commitment to, and effectiveness in, achieving its targets and, hence, anchor inflation expectations. This is because economic agents are expected to have confidence in the effectiveness of monetary policy rate in keeping inflation stable. It is therefore important because perception on inflation expectation differs among different categories of economic agent due to the heterogeneity in the composition of their consumption basket and how inflation affects them differently. If inflation expectation is not firmly anchored on monetary policy rate and inflation expectations is in disarray,
inflation persistence would rise as the gap between actual and expected inflation widens, which dampens the chance of hitting the desired inflation target. Thus, the desire to attain price stability in the long-run relies heavily on the credence given in the management of expectations because it goes into the heart of the inflation targeting framework. Bearing in mind that price stability is the key mandate of the Central Bank of Nigeria (CBN), it becomes even more important to the CBN to aim at it targets accurately.

Another fundamental issue in the conduct of monetary policy, especially when managing inflation, is the common pitfall in how the central bank models the relationship between inflation and output. This is critical for optimal monetary policy, particularly when the inflation-output relationship exhibits nonlinearities and is not taken into consideration. A possible effect of ignoring these non-linearities is that policy decisions would be bias on inflation. Therefore, the possibility of achieving disinflation becomes increasingly challenging. Furthermore, when regime change occurs within inflation horizon and monetary authority fails to react appropriately, policy variable could become less responsive to the nominal anchor. Under the current framework of the CBN, this could lead to loss of credibility and may also be detrimental to the success of monetary policy in attaining price stability no matter how transparent the Bank may be.

Despite these concerns, the bulk of existing studies on the dynamics of inflation continue to have fundamental methodological flaws. For instance, several recent studies in developing economies that have attempted providing empirical evidence on inflation dynamics simply assumed away the potential non-linearities highlighted above. These studies, for example, include Bawa, Abdullahi and Ibrahim (2016) for Nigeria; Addo (2016) for Ghana; Kulatunge (2017); Manopimoke (2018); and Bekiros et al. (2017) for Sri Lanka, Thailand and other Asian economies, respectively. Each of these studies reported evidence of dynamism in the inflationary process in each of these countries, albeit within a linear framework. The linearization of the New Keynesian Philips Curve (henceforth, NKPC) has its own problems. Instead of estimating the NKPC in its nonlinear form, linearizing it may lead to substantial loss of information because of the restriction of the autocovariances of the residuals, which are assumed to be constant. This restriction strips away
the information laden in the residuals, which could adversely affect the forecasting ability of the model.

In view of the issues raised, this paper seeks to estimate a non-linear open-economy NKPC on a disaggregated, rather than aggregated, CPI for Nigeria. Specifically, the paper aims to answer the following questions. First, are there nonlinearities in the NKPC for Nigeria? Which of the sub-component of the consumer basket dominates the changes in the aggregate consumption basket? What are the drivers of domestic inflation in low and high regime? What is the inflation threshold that triggers regime switch? What is the magnitude and speed of exchange rate pass-through during the study period under review? What is the behavior of inflation (disinflation) across the regimes? In addition, because the open-economy feature in the NKPC is accounted for by introducing import prices and exchange rate, the paper also estimates the exchange rate pass-through to domestic inflation.

The rest of the paper is organized as follows. Section two provides a review of literature, section three is the methodology which entails the theoretical framework, specification of the model and data used. Section four is the presentation and discussion of results. Section five concludes the study.

2.0 Literature Review

In analyzing inflation dynamics. The wide application of the of the New Keynesian Philips Curve (NKPC) framework in many recent studies attest to the adequacy and relevance of the model in explaining inflation dynamics. Some of the early empirical studies that embraced the framework include Gali, Gertler and Lopez-Salido(2005), Cogley and Sbordone (2006), Cecchetti et al. (2007), Fuhrer and Olivie (2010) among others.

The baseline NKPC which provides explanation of changes in inflation based on changes in the output gap and the effect of forward looking inflation as a measure of expected inflation is documented in Zhang (2014), Mavroeidis, Plagborg and Stock (2013), Fuhrer, Olivie and Tootell (2012), Gertler and Leahy (2008), Zhang, Osborn and Kim (2008). Their estimate of the model showed that output gap as measured by the detrended output explains inflation dynamics within the NKPC. However, earlier study by Cogley and Sbordone (2005 and 2006) argued that
inflation dynamics associated with backward looking process and derived from long-run trend component of inflation, must incorporate the interaction between the drift in trend fluctuation and nonlinearity. This argument draws attention to the importance of nonlinearity in the NKPC and the need to specify the model as an AR-process.

In examining the inflation dynamics and forecast of the U.S, the study by Brissimis, Nicholas and Magginas (2008) provides strong evidence in support of NKPC in explaining inflation dynamics. Their comparisons of the NKPC and the hybrid version however showed that the estimate of the hybrid NKPC was weaker. Similarly, Zhang, Osborn and Kim (2009) further buttress the success of the NKPC in explaining inflation dynamics in the U.S using the baseline theoretical exogenous variables.

More recent empirical evidence that centered on open economy NKPC was presented for G-7 countries in Lagoa (2014). The study reveals that cost channel as measured by import prices is essential in open economies and this plays an important role in explaining inflation dynamic in an open economy using NKPC. The case of other developed economies, Kichian and Rumler (2014) estimate both open and closed versions of the NKPC for the Canadian economy. They argue that single equation structural model is a strong option in explaining inflation dynamics and forecast. However, other studies such as Melolinna (2015) used a non-structural model (Factor-Augmented Vector Autoregression framework) in explaining the factors that are responsible in explaining inflation dynamics for United Kingdom and Euro area. The study contends that while demand side shocks are responsible in explaining inflation dynamics there is evidence, of exchange rate effect in driving changes in the headline inflation in the countries. This study, therefore, underlines the importance of the open economy version of the NKPC especially for small open economy.

Empirical evidence of inflation dynamics from developing economies were reported in Bawa, Abdullahi and Ibrahim (2016) and Addo (2016) for Nigerian and Ghanaian economy respectively. Both studies highlight evidence of persistence and the role of exchange rate in explaining inflation dynamics. Earlier study on Africa in Ayubu (2013), incorporates exchange rate in explaining inflation in the case of Tanzania. The case of Asia and Middle-east were reported in Kulatunge (2017) who documented evidence for Sri Lanka, Manopimoke (2018) report for
Thailand and Almounsor (2010) for case of Yemen. Others are Bekiros et al. (2017) who provided a cross country evidence from Asia. All of the Asian and Middle-east evidences suggest exchange rate as a strong factor in explaining inflation dynamics. This strand of literature, therefore, reinforces the evidence that inflation dynamics in developing economies are better explained by external shocks that are transmitted through exchange rate value of local currencies of these countries. More so, in the case of Thailand and Nigeria, the structural single equation appears to be promising in explaining inflation dynamics (see Bawa et al., 2016 and Manopimoke, 2018).

Empirical evidence on inflation dynamics in Nigeria were reported in Audu, Yaaba and Ibrahim (2018), Modebe and Ezeaku (2016), Tule et al. (2015). Although, these studies reveal the behavior of inflation and explicitly explain how persistent inflation dynamics occur in Nigeria, the emphases are more on the role of money supply in explaining the inflationary environment. No relevance was given to the open nature of the economy. Thus because of the degree of openness and prominence the CBN gives to exchange rate, it is imperative to consider exchange rate in explaining inflation in Nigeria. In addition, the atheoretical framework of VAR was used, which suffers from lack of micro foundation and it is policy invariant. Other study such as Odo, Odionye and Ojike (2016) provide a disaggregated approach in explaining inflation dynamics in Nigeria. The evidence suggests that food inflation is a dominant component of the consumer basket that accounts for inflation in Nigeria. However, this study equally ignored the role of exchange rate.

The application of NKPC in explaining inflation dynamics in Nigeria is reported in Garcia (2010) and Rasaki (2017). Using general equilibrium framework, Garcia (2010) observes that the NKPC is relevant in explaining inflation in Nigeria. Rasaki (2017) tries to augment the Philips curve with money supply, the study only considers foreign input in accounting for the openness and ignores exchange rate. In addition, the estimate is only an approximation of the Philips curve because it does not factor-in the nonlinear structure of the Philips curve and does not lend support to Chuku, Atan and Obioesio (2017), who provide evidence of structural break in inflation dynamics in Nigeria. Other applications of the

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4 See Bello and Sanusi (2018) on the behaviour of Monetary Policy Committee in Nigeria
Philips curve in explaining inflation in Nigeria include Jelilov and Balewa (2018), Isah and Joel (2018), Orji, Orji and Okafor (2015), Onakoya (2015), Ogujiuba and Abraham (2013), Ojapinwa and Esan (2013), Iyeli and Ekpung (2017). This strand of literature estimates the Philips curve using autoregressive distributed lag model. The application of cointegration approach in these studies is somewhat inconsistent with the Friedman’s theoretical postulate of a vertical Long-run Philips curve\(^5\). There has been proliferation of literature on exchange rate pass-through to domestic inflation in recent times. Most studies observed similar findings over varying periods. For instance, studies such as Iyoha (2017), Musa (2017), Bada et al. (2016), Chuba (2015) and Zubair, Okorie and Sanusi (2013) report that the exchange rate pass through to domestic prices is low and incomplete for Nigeria. In addition, the evidence suggests that the speed of the pass-through is relatively slow. In line with this finding, Adekunle and Ajao (2018) reports that exchange rate pass-through in Nigeria is incomplete. However, the study contends that pass-through to domestic prices increasingly becomes larger particularly when the asymmetric effect of exchange rate is considered. The study also emphasizes the relevance of adaptive expectation in explaining inflation in Nigeria. It is worthy to note that the divergence between this evidence with earlier studies is because of differences in the way inflation is modelled. This means there is a substantial difference in the size of the exchange rate pass-through when inflation is approximated through linearization and when other sources of nonlinearity are considered (see Chuku, Atan and Obioesio, 2017)

In the light of the reviewed literature, four fundamental lessons are taken. First, the NKPC provides a suitable framework for explaining, as well as forecasting, inflation dynamics. Secondly, the baseline exogenous variables are not sufficient in explaining inflation dynamics in small developed economies as well as in developing economies\(^6\). Thirdly, incorporating persistence is critical in understanding inflation dynamics especially in developing economies (see Rasaki, 2017 and Adekunle and

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\(^5\) Friedman 1968 argues that the long-run relationship between inflation and unemployment does not exist because the Philips curve is vertical in the long-run. Therefore, the long-run Philips curve is at a position of Non-accelerated Inflation Rate of Unemployment (NAIRU) which coincides with the natural rate of unemployment.

\(^6\) This provides justification for incorporating other exogenous variables in NKPC.
Ajao, 2018). Lastly, the dominant approach used in the estimation of NKPC is the linearized methods as reported in Jelilov and Balewa (2018), Isah and Joel (2018), Iyeli and Ekpung (2017), Iyoha (2017), Musa (2017), Bada et al. (2016), Chuba (2015), Orji, Orji and Okafor (2015), Onakoya (2015), Ogujiuba and Abraham (2013), Ojapinwa and Esan (2013), and Zubair, Okorie and Sanusi (2013). The assumption of linearity is, however, too strong for a number of reasons. First is that, both the traditional and New Keynesian Philips Curves are known to be structurally nonlinear. Therefore, linearization of the NKPC will only serve as a lower approximation of the parameters. Second, the use of linearized approach will not account for monetary policy shifts, especially in terms of accommodation (non-accommodation) of shocks. This is very important in distinguishing low-inflation environments from high-inflation environments. In addition, linearization will not illuminate those among the monetary policy targets to be prioritized when faced with conflicting monetary policy objectives.

3.0 Data and Methodology

This section focuses on establishing the framework used for the study. The empirical model was drawn from the theoretical framework as documented in the literature. The empirical specification of the open-economy NKPC was augmented based on the open nature of the Nigerian economy. In addition, the disaggregated consumer basket is specified to enable identification of the sub-components that exert greater weight in the consumption basket.

The main criticism of the NKPC framework relates to its purely forward-looking features, it does not imply persistence of inflation. How was this addressed in the paper?

3.1 Data Description

The data used in our empirical augmented Philips curve were drawn from the Central Bank of Nigeria (CBN) statistical bulletin and all variables are quarterly data spanning from 1995Q1 to 2018Q2. The limit of the period under review was determined by the availability of data as at the period the study was conducted. The Consumer Price Index (CPI) was used as the measure of inflation, which was further disaggregated into food inflation (both imported food and farm produce) and core inflation. Food inflation represents the share of the CPI that is from farm produce while
the core inflation, were energy is a dominant component is used as proxy for energy inflation. The aggregate import price was computed as the ratio of nominal to real imports. The real marginal cost of firms as derived in the hybrid NKPC is proxied by compensation of employees (in billions of naira) at 1990 constant prices (CBN statistical bulletin, 2018). The data on exchange rate is measured as exchange rate value of the naira to the U.S dollar (Naira/U.S Dollar). Figure-1 is the plots of the data set showing the behaviour of the trend of variables over time.

Figure 1: Evolution of the variables

3.2 Theoretical Model
In stating the theoretical NKPC, the consumer basket is expressed as composite and being explained by firms’ cost (marginal cost) of producing the items that goes into the consumer basket. This is express as:

\[
\pi_t = \frac{(1 - \theta)(1 - \theta \alpha)((mc_t - p_t) + \rho)}{\theta} + \alpha E_t \pi_{t+1}
\]

where \( \rho \) is the fixed markup, \( mc_t \) is the firms marginal cost, \( \theta \) is the coefficient that captures price stickiness, \( E_t \) is expectation, \( \pi_t \) is the current inflation and \( \alpha \) represents the weight firms place on future losses relative to current losses due to stickiness.
Eq. (1) is referred to as the NKPC. It is worthy to note that inflation is determined not by nominal marginal cost \((mc_t)\) but by real marginal cost \((rmc_t)\), we therefore have

\[
rmc_t = mc_t - p_t
\] (2)

The importance of eq. (2) rests on the fact that the disparity between the nominal marginal cost and the current price is what induces inflationary tendencies. Especially as \(mc_t \gg p_t\), a frictionless Calvo firm will review prices upward persistently until the increasing cost due to additional factor input declines from either wages or interest on capital. Therefore, we have the NKPC as:

\[
\pi_t = \frac{(1 - \theta)(1 - \theta \alpha)(rmc_t)}{\theta} + aE_t \pi_{t+1}
\] (3)

Now we proceed by adapting the model to allow the decomposed consumer basket to take prominence.

### 3.3 Model Specification and Econometric Issues

The simplified version of the NKPC model augmented with the disaggregated consumer basket is specified as follows:

Starting with theoretical NKPC of the following form

\[
\pi_t = \phi^f E_t (\pi_{t+1}) + \phi^b \pi_{t-1} + \beta rmc_t + \epsilon_{\pi t}
\] (4)

where \(\phi^f\) and \(\phi^b\) are the forward and backward\(^7\) looking coefficient of inflation respectively. Adding the three disaggregated components of consumer inflation, therefore, yields the disaggregated model:

\[
\pi_t = \beta_0 + \phi_1^f E_t \pi_{t+1} + \phi_2^b E_t \pi_{t-1} + \phi_3 \pi_e + \phi_4 \pi_f + \vartheta \pi_{imp} + \beta rmc_t + \lambda exr_t + \epsilon_{\pi t}
\] (5)

Where \(\pi_f\) and \(\pi_e\) represent food and energy inflation while \(\beta_0\) is the drift parameter of the Philips curve. Because of the open structure of the underlying economy, inflation can be imported \((\pi_{imp})\) to domestic economy. Hence trade with rest of the world is invoiced in US dollar, therefore the exchange rate \((exr_t)\) is introduced to capture relative change

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\(^{7}\) This also represents the persistence in inflation following Bello et al (2016).
in the exchange rate value of the naira. $\epsilon_{\pi t}$ is the stochastic error term that account for the disturbances.

This paper explores the possibility of non-linearities in the Philips curve after augmenting the curve with import prices and exchange rate. This possibility is plausible in the open economy model because, as found in Bello and Sanusi (2016), the Nigeria’s monetary authority reacts to exchange rate value of the naira when there are large swings in currency depreciation to stabilize the local currency (Naira). The moderate and large fluctuations in the currency, therefore, provide the basis for alternative regimes in the reactions of the monetary authorities to inflation threats. These regimes switching tendency is particularly observed in monetary policy environment, such as Nigeria, where the monetary policy seem to manages the announced exchange rate within a given range (see Monetary Policy Compendium, 2013). This position aligns with Chappell et al (1996) as also found in Franses and Paap (2000) and Franses and van Dijk (2003).

In addition, the possibility of non-linear adjustment is explored, because linear models are less useful in out of sample forecasting (Franses and van Dijk, 2000). The key drawbacks of the linear model in forecasting follows from the properties of the white noise, defined as:

\[ E(\epsilon_t) = 0 \]
\[ E(\epsilon_t^2) = \sigma^2 \]
\[ E(\epsilon_t \epsilon_p) = 0 \quad \forall \ t \neq p \]

The justification that $E(\epsilon_t \epsilon_p)$, which are the autocovariances, are equal to zero signifies the incapability of the white noise in the linear model to contain information set in forecasting future inflation.

In this study, a nonlinear modeling of inflation simply connotes defining inflation in different states or regimes. By extension, dynamic behaviour of inflation is determined by the regimes that exist in the series, as noted in Frances and van Dijk (2000) and Priestly (1980). Similarly, this means

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8 Though the exchange rate pass-through is not pre-occupation of this paper but it provides some impetus in explaining the effect of the open nature of a small open economy.
that the mean, variance and the autocovariances (or autocorrelation) also differ between regimes.

3.4 Empirical Specification of Augmented NKPC

We specify the derived Philips curve in the spirit of Terasvirta (1992) who allow exogenous variable in the nonlinear specification:

\[ \Delta \pi_t = (\beta_0 + \varphi_1 E_t \pi_{t+1} + \varphi_2 E_t \pi_{t-1} + \varphi_3 \Delta \pi_e + \varphi_4 \Delta \pi_f + \vartheta \Delta \pi_{imp} + \beta \Delta rmc_t + \lambda \Delta e^x_t)G(s_{t-1}, \gamma, c) + \epsilon_{\pi t} \]  

(6)

where \( G \) is the nonlinear transition function which could be replaced with an indicator function \( I[\pi_{t-1} > c] \) if the transition variable test indicates the endogenized variable as the transition variable\(^{10} \) otherwise it is continuous function which changes smoothly from 0 to 1. \( s_{t,i} \) is the transition variable, which will determine if the Philips curve will be a Self-Exciting Threshold Autoregressive model (henceforth SETAR) or Smooth Transition Autoregressive model (henceforth, STAR)\(^{11} \), \( \gamma \) measures the speed of transition function or slope\(^{12} \) of the transition, which measures the change of \( G(s_{t-1}, \gamma, c) \) from zero to one (0 to 1) as suggested in Van Dijk, Terasvirta and Frances (2002). \( c \) is the threshold value or location parameter that triggers nonlinearity in the Philips curve. That is the threshold between the two aspects of the NKPC determined by the nonlinear part \( [G(s_{t-1}, \gamma, c) = 0 \text{ and } G(s_{t-1}, \gamma, c) = 1] \).

Equation 6 can be referred to as the semi parametric nonlinear model because it bears an AR(1) component which is both parametric and linear. In addition, it has a nonparametric and nonlinear component, which also determines the regime switch. The explanatory variables that enter into

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\(^9\) Note that the empirical specification of the NKPC in equation 5 & 6 is a hybrid form of the model. This is including backward-looking behaviour, which addresses the issue of inflation persistence.

\(^{10}\) Once the transition variable that accounts for nonlinearity in the Philips curve is attributed to the lag inflation then this means that the nonlinearity is self-excited (SETAR) by inflation itself. When this happens, the nonlinearity is sharp or sudden curve but if not, a STAR will result in a smooth nonlinear Philips curve.

\(^{11}\) This depends on whether the transition variable is found to be the dependent variable of an exogenous variable.

\(^{12}\) Note that the magnitude of \( \gamma \) determines the shape of the non-linear Philips curve. If \( \gamma = 0 \) the Philips curve will be linear, if \( \gamma < 25 \) it will be a logistics or smooth transiting curve and when \( \gamma > 25 \) the Philips curve will be a self-exciting curve with a clear perpendicularity. However, this is only a necessary condition, the sufficient condition for determination of either the nonlinearity is a SETAR or STAR is confirmed by the endogenized variable.
equation 6 are: adaptive expectation \((\pi_{t-1})\) which accounts for persistence inflation due to backward looking economic agent, energy component of inflation \((\pi_{e})\) this explains how price setters adjust prices in the goods market, food inflation \((\pi_{f})\) this captures general price behaviour of both domestic and imported food, imported prices of raw material, intermediate and final goods \((\pi_{imp})\), real marginal cost \((rmc_t)\) this explains the wage setting behaviour in the labour market and lastly is the exchange rate of the domestic currency \((exr_t)\) this captures the relative price of the domestic currency to that of the US dollar.

Therefore, following Bello and Sanusi (2017) we proceed with determination of the transition variable that is responsible for the nonlinear switch and to establish if the nonlinear model is SETAR or STAR. Also, it is imperative to undertake a linearity test to establish the specific variant (either first or second order Logistics Smooth Transition Regression, i.e., LSTR1 or LSTR2) of the model to be estimated if the transition variable confirms STAR.

Thus, following Kratzig (2005) the transition variable \(s_{t,i}\) is determined using the auxiliary equation:

\[
y_t = \beta_0' z_t + \sum_{j=1}^{3} \beta_j' \tilde{z}_t^j s_t^j + u_t^* \tag{7}
\]

Where \(z_t\) is the vector of explanatory variables with the potential transition variable not inclusive, \(\tilde{z}_t\) is the vector of explanatory variable with the potential transition variable inclusive, \(\beta_j'\) is coefficient of all the potential transition variable can take in the test equation, \(y_t\) is the notation of the dependent variable \((\pi_t)\) in the test equation and \(u_t^*\) is the residuals of the estimated test equation.

The choice of the model was made based on the following hypotheses:

\[H_{04}: \beta_3 = 0\]
\[H_{03}: \beta_2 = 0 | \beta_3 = 0\]
\[H_{02}: \beta_1 = 0 | \beta_2 = \beta_3 = 0\]

It is worthy to note that in each of the null hypotheses \((H_{04}, H_{03}\) and \(H_{02}\)) decision rule is determined from the computed F-statistical values: \(F4, F3,\)
F2 and the overall F-value. Hence the element in \( \tilde{z}_t \) that corresponds with the minimum F-statistical values represents the transition variable and the transition function.

4.0 Results and Discussion

As shown in Table 1, the mean of exchange rate of the domestic currency is greater than the mean of all the various components of the consumer inflation as well as mean of the firms’ real marginal cost \((rmc_t)\). The high relative value of the U.S dollar to the domestic currency justifies the relative high mean value of the rate of exchange \((exr)\). Based on the reported standard deviation, the exchange rate shows more volatility than other variables in the model. As for the various measures of inflation, both food price inflation \((\pi_f)\) and energy inflation \((\pi_e)\) converges towards the same mean value.

Table 1: Result of Unit Root Test

<table>
<thead>
<tr>
<th>Variable</th>
<th>Level</th>
<th>1st diff.</th>
<th>Descriptive stat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi_t)</td>
<td>-5.539(0.000)*</td>
<td></td>
<td>91.30 64.6 0.85 94</td>
</tr>
<tr>
<td>(\pi_e)</td>
<td>-3.557(0.039)*</td>
<td></td>
<td>91.29 62.5 0.74 94</td>
</tr>
<tr>
<td>(exr_t)</td>
<td>2.027(0.578)</td>
<td>8.897(0.000)**</td>
<td>144.34 97.3 1.36 94</td>
</tr>
<tr>
<td>(\pi_f)</td>
<td>-2.302(0.428)</td>
<td>-8.819(0.000)**</td>
<td>93.72 66.5 1.01 94</td>
</tr>
<tr>
<td>(\pi_{imp})</td>
<td>-2.048(0.566)</td>
<td>-13.405(0.000)**</td>
<td>21.71 7.7 0.12 94</td>
</tr>
<tr>
<td>(rmc_t)</td>
<td>-5.613(0.000)*</td>
<td></td>
<td>28.76 14.8 0.12 94</td>
</tr>
</tbody>
</table>

* and ** indicates stationarity at levels and 1st difference respectively using 95% level of significance. Source: Authors computation. \( \mu = \text{mean} \) & \( \sigma = \text{standard deviation} \)

The preliminary test begins with Philip-Perron unit root test to determine if the variables are non-stationary. The result shows that all the variables are not stationary. The test result is consistent with inflation data in many countries (for example U.S., England, Germany, Australia, etc.).

We proceed with determination of the appropriate lag length of the nonlinear model and the specification strategy is consistent with the suggestion by Box and Jenkins (1970).

The optimal lag is critical in determining structure of the AR-process that make up the low inflation environment. In addition, where the possibility of having multiplicity of the transition variable and more than one regime switch exist, variety of lags (variables) offer the benefit of doing so
without fear of encountering the problem of autocorrelation. Hence the result of the optimal lag length is presented.

Table 2. Result of Optimal Lag Length Selection

<table>
<thead>
<tr>
<th>Lag</th>
<th>LogL</th>
<th>LR</th>
<th>FPE</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
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<td>2.40e+10</td>
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<td>839.0725</td>
<td>194320.1*</td>
<td>29.20098</td>
<td>30.51878*</td>
<td>29.72614*</td>
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<td>29.50212</td>
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<td>384462.0</td>
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<tr>
<td>4</td>
<td>-939.6514</td>
<td>45.41576</td>
<td>442064.1</td>
<td>29.85346</td>
<td>34.55989</td>
<td>31.72905</td>
</tr>
<tr>
<td>5</td>
<td>-891.5961</td>
<td>55.29656</td>
<td>377133.5</td>
<td>29.52318</td>
<td>35.35914</td>
<td>31.84891</td>
</tr>
<tr>
<td>6</td>
<td>-852.9742</td>
<td>38.09279</td>
<td>462158.1</td>
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</tr>
<tr>
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<td>-780.8900</td>
<td>59.24732*</td>
<td>263376.5</td>
<td>28.46274*</td>
<td>36.55779</td>
<td>31.68876</td>
</tr>
</tbody>
</table>

Source: Authors computation, 2019

As shown from Table 2, the sequential modified log-likelihood ratio (LR) test statistics and Akaike Information Criteria (AIC) selected lag 7 as the optimal lag length. However, the Final Prediction Error (FPE), Schwarz Criteria (SC) and Hannan-Quinn (HQ) selected lag length 1 as the optimal lag length. Given the size of the data set available and having majority of the selection criteria to have selected lag 1 as the optimal lag length, therefore we proceed with lag 1.

In testing for the presence of nonlinearity, all variables were given equal chance of being selected as the transition variable. As such no restriction was imposed on any of the exogenous variables. This allows the empirical determination of the actual “cause” of the differences in the dynamics of inflation. The results of the test are presented in Table 3, which shows that exchange rate is identified as the transition variable ($s_{t,i}$) and is an element of $z_t$ in equation 7. This clearly identifies the level of exchange rate depreciations (high vs low) as the regime separator in the dynamics of inflation in Nigeria. The results of the nonlinearity test are presented as NaN (Not a Number, in column 6) in Table 3. Hence, because the test regression presented in equation 7 and the power of the transition variable is included, this raises the problem of invertibility when the $s_{t,i}$ candidates are close to 0 or 1.
Inflation Dynamics and Exchange Rate Pass-Through in Nigeria: Evidence from Augmented Nonlinear New Keynesian Philips Curve

As shown in Table 3, exchange rate is established as the transition variable because it has the lowest F-statistics in $F$, $F4$ and $F3$ column (exception of $F2$). We reject the null of linearity against non-linearity which falls in the category of the exponential smooth transition variant of the STAR model. This suggests that the semi-parametric nonlinear Philips curve is not a logistics STAR, but an exponential variant of the STAR model. This mean that the shape of the nonlinear NKPC will not change monotonically but rather exponentially in response to the transition variable.

Having identified the transition variable and the structural form of the NKPC model, we performed a grid search since the hypothesis of linearity was rejected against STR nonlinearity. For the grid search, to ensure that the initial values used for the estimation are scale-free, we followed Bello and Sanusi (forthcoming), to divide $\gamma$ by the $K^{th}$ power of the sample standard deviation of the transition variable.

Overall, the results of the nonlinearity tests, so far, suggest that the dynamics of inflation in Nigeria is non-linear with distinct regimes. These regimes are demarcated by the level of exchange rate depreciation (the threshold variable). The characteristics of the dynamics depend on the extent of exchange rate depreciation (see Table 4a and 4b).

Next, we present the estimated non-linear NKPC model as presented in equation (6). In doing so, the general unrestricted version of the hybrid model was estimated, in which all the theoretically relevant variables were included. However, the coefficients of the forward-looking variables were not statistically significant. In addition, the magnitude of the estimated coefficient on the transition variable is not plausible. The estimated results of the unrestricted hybrid NKPC is shown in the Appendices I and II.

### Table 3: Result of Linearity Against Nonlinearity STR

<table>
<thead>
<tr>
<th>Transition Variables</th>
<th>F</th>
<th>F4</th>
<th>F3</th>
<th>F2</th>
<th>Suggested Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{t-1}$</td>
<td>1.684e-06</td>
<td>1.611e-01</td>
<td>1.453e-03</td>
<td>1.378e-05</td>
<td>LSTR1</td>
</tr>
<tr>
<td>$\pi_{t+1}$</td>
<td>8.259e-08</td>
<td>1.8010e-01</td>
<td>4.695e-01</td>
<td>2.5799e-04</td>
<td>LSTR1</td>
</tr>
<tr>
<td>$\pi_e$</td>
<td>1.228e-06</td>
<td>2.114e-01</td>
<td>8.270e-04</td>
<td>1.108e-05</td>
<td>LSTR1</td>
</tr>
<tr>
<td>$exr^*_{t}$</td>
<td>7.554e-08</td>
<td>2.984e-02</td>
<td>7.349e-05</td>
<td>9.036e-05</td>
<td>LSTR2</td>
</tr>
<tr>
<td>$\pi_f$</td>
<td>5.294e-06</td>
<td>1.470e-01</td>
<td>5.845e-03</td>
<td>1.427e-05</td>
<td>LSTR1</td>
</tr>
<tr>
<td>$\pi_{imp}$</td>
<td>6.250e-06</td>
<td>5.601e-01</td>
<td>6.561e-02</td>
<td>6.349e-08</td>
<td>LSTR1</td>
</tr>
<tr>
<td>$rmc_t$</td>
<td>1.736e-04</td>
<td>4.225e-01</td>
<td>6.201e-03</td>
<td>2.530e-04</td>
<td>LSTR2</td>
</tr>
<tr>
<td>trend</td>
<td>1.961e-05</td>
<td>7.587e-01</td>
<td>5.232e-03</td>
<td>2.081e-06</td>
<td>LSTR1</td>
</tr>
</tbody>
</table>

Source: Authors computation, 2019
most parsimonious version of the model was re-estimated, which turns out to have no forward-looking components, are presented in Tables 4a and 4b, for interpretation\textsuperscript{13}. The restricted backward-looking model is consistent with Ronderos (2015).

4.1 Low Exchange Rate Depreciation Regime (Low Inflation Environment)

The first regime as captured by an AR process of the NKPC constitutes the AR-part that is almost parallel and asymptotic to the horizontal axis of the cartesian plane. The curve assumes restricted intercept because even when inflation is very low it may never return to zero. Hence the range of the data does not include more than one base period\textsuperscript{14}.

As shown in Table 4a, at low inflation environment\textsuperscript{15} both food and energy prices have significant influence on the average changes in the composite consumer basket. However, food inflation has greater influence on the general inflation than any of the other component of inflation. About 79.8\% of the variation in the consumption basket prices is accounted for by general changes in food (both imported and domestically produced) prices. This means that much part of the changes (increase) in general prices will stem from changes in either domestically produced food prices or imported food prices or as a result of both. Therefore, factors such as poor harvest due to bad weather or increase in cost of farm input can be considered as contributing factors to food inflation. To a lesser extent domestic energy inflation accounts for about 57.0\% of the price changes in the total consumption basket. The domestic factors that leads to changes in energy cannot be far-fetched from the instability in supply\textsuperscript{16} (shortage) of Premium Motor Spirit (PMS) and Automotive Gas Oil (AGO).

\textsuperscript{13} Note that in line with empirical evidence reported in Rasaki (2017) and Adekunle and Ajao (2018) who observed that the estimate of expected inflation is insignificant and low in explaining inflation in Nigeria, equation 6 is therefore restricted with rational expectation parameter for $E_t \pi_{t+1}$ taken to be zero

\textsuperscript{14} Where base period overlap exists, such series are converted to reflect the initial base period. In this case data are drawn based on 1990 base period.

\textsuperscript{15} This is an environment where general prices are relatively stable, hence inflation is under the control of monetary authority.

\textsuperscript{16} This are often perennial changes in oil regime when government subsidy payment to marketers are unpaid over long time.
This finding has an important implication on macroeconomic management strategy in Nigeria. That is, domestic inflationary pressure is often driven by supply side factors especially when the economy is in a relative low inflation environment. Therefore, monetary authority in Nigeria must not react by tightening the monetary policy rate in order to forestall changes in inflation that are caused by supply factors (cost push inflation). Thus, monetary policy would be effective only to the extent that these key components are responsive to demand-side factors. Because food and energy prices are more of supply side factors, the nominal anchor may not appropriately be able to keep these prices in check. Therefore, the implication of tightening the monetary policy rate in response to changes in the average price of aggregate consumption basket may be less effective compared to when policies are targeted at the supply factors. Hence in its effort to adopt inflation targeting framework, the CBN should endeavor to target what it can hit (which may require constructing some dedicated measures when supply or cost push inflation hits the economy). For instance, by guiding money market into discriminating the cost of borrowing\textsuperscript{17} depending the intended target of the central bank.

The result also indicates that about 8.1\% of the domestic inflation is attributed to imported inflation. This means that changes in prices of foreign commodities increases domestic inflationary pressure. This is the so-called second-stage pass-through, in line with Zubairu, Okorei and Sanusi (2013), who argue that it is expected to be smaller because of a

\textsuperscript{17}Charging discriminatory lower interest rate to specific targets (sector or industries) in order to lower the marginal cost of production. This can mitigate the effect of negative supply shock.
number of reasons including domestic competition (largely explained by the theory of pricing to market) and significant domestic value addition. Therefore, it is expected that the magnitude of the effect of changes in the composite consumer basket that is caused by import prices is relatively negligible and has the least influence in the basket (see Sanisu, 2012). This conforms with the theory of pricing to market followed by exporting (importing) firms.

In this regime, the result equally shows an evidence of less persistence in inflation in the low inflation regime. The less persistence (indicated by the negative coefficient of past changes in aggregate consumption basket) that characterizes inflation in this regime, substantiates the argument laid down about the driving force of inflationary pressure in low inflation environment. These are indeed supply side shocks rather than demand shocks. This provides some indication of achieving disinflation in the low exchange rate depreciation regime.

The coefficient of real marginal cost, which is the major determinant of cost push inflation indicates that about 18% of the changes in the composite consumption prices in Nigeria is as a result of increase in marginal cost of firms. These changes could be due to increase in firms’ wage bill of employees. In terms of domestic output, in low inflation regime, the moderate effect of the marginal cost suggests that output gap is relatively wide. The wide output gap implies existence of slack in the labour market. This signifies high unemployment rate. Therefore, there will be little or no rise in firms’ nominal wage bill because, when unemployment rate is high, nominal wages do not rise as much. This means that actual output is by some measure far away from its potential level. This strongly reinforce the behavior of inflation relative to output in low inflation regime. That is, with some slack in the labour market, the economy will be in low inflation regime and cost push effect will be moderate. Meanwhile the evidence of inflation inertia is believed to be stemming from the price setting behavior of the representative firms in the economy. More so, these firms might set both their mark-up and prices based on past prices. Also, the inflation inertia could be attributed to wage demand of worker and wage setting of the employers. This means that

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Note that $fim's price(P) = Nominal wage bill(W) + markup$

Recall that the nominal wage is determined by $W = P^e(u, \lambda)$
employers tend to set current wages with some friction relative to past wages. The trade–off between inflation and unemployment is very low and insignificant probably because monetary policy authority in Nigeria are more predisposed to fighting inflation by keeping the MPR high, than loosening to allow rise in output. This finding is consistent with the primary mandate of CBN (see Bello and Sanusi 2016, 2018).

One of the key finding for policy is that monetary authority can achieve disinflation when inflation regime is characterized by low persistence in inflation in the presence of a low exchange rate depreciation. (The main criticism of the NKPC framework relates to its purely forward-looking features, it does not imply persistence of inflation. How was this addressed in the paper? See Section 3.0) Overall the shape of the Philips curve under this regime is relatively flat. This means unemployment can be reduced without having higher inflation.  

4.2 High Exchange Rate Depreciation Regime (High Inflation Environment)  
In switching from low regime to high exchange rate regime, the result of smooth transition regression model indicates that NKPC switches smoothly as it follows an exponential transition shape. The result presented in Table 4b shows that inflation switches to a higher regime at certain threshold value. Inflation threshold represents a specific level of inflation which if taken by exchange rate, the NKPC exits from the low inflation regime to high inflation regime. This is referred to as the inflation threshold of regime switch and it was found to be about N75. This means that a depreciation (devaluation) that leads to decline in the value of the naira by as much as N75 will trigger a regime switch from low to high inflation regime. Therefore, the result of the threshold suggests that a decline in the value of the domestic currency by about N75 against the US dollar (which might be due to depreciation or devaluation of the currency) has high propensity of 99 per cent to triggers a regime switch from low inflation to a high inflation environment. 

Thus, the result in Table 4b indicates that the speed with which exchange rate depreciation (relative to U.S dollar) will pass-through to inflation will be 70 per cent quarterly until all the inflationary pressure that accumulate

20 One way of achieving this is to pursue supply side policies such as research and development especially in the agricultural sector.
because of the depreciation dissipates over this horizon. Note that
dissipation is completed only when the inflation has switch completely to
high inflation regime. Hence exchange rate appears to be very critical in
explaining not only different inflation regime, but also inflationary
process in Nigeria. More so, empirical evidence from the result have
shown that exchange rate is the major determinant of inflation dynamics
in a high inflation environment.

Table 4b: Result of Augmented Philips Curve and Exchange Rate Pass-
Through in High Inflation Regime

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ψ_{2}</td>
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</tr>
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<td>beta_{2}</td>
<td>-0.708</td>
<td>0.0000</td>
</tr>
<tr>
<td>lambda_{2}</td>
<td>0.0401</td>
<td>0.0420</td>
</tr>
<tr>
<td>y</td>
<td>-0.703</td>
<td>0.0045</td>
</tr>
<tr>
<td>c1</td>
<td>74.97</td>
<td>0.0084</td>
</tr>
<tr>
<td>c2</td>
<td>74.97</td>
<td>0.0084</td>
</tr>
</tbody>
</table>

Note: ** and * indicates level of significance at 5% and 1% respectively.

The coefficient of exchange rate is found to be statistically significant and
shows that only 4.2% depreciation in the naira pass-through to domestic
inflation after the switch. This means that 4.2% of the domestic
inflationary pressure comes from a rise in the relative value of the dollar
against the naira but this occurred in high inflation regime. This means, if
exchange rate instability is not tackled then small changes in the value of
naira against the dollar will continue to pass-through to domestic inflation.
However, this pass through to domestic inflation is slow and incomplete.

This paper finds the critical role exchange rate plays in the Nigeria’s
inflationary process. First, the exchange rate, being the transition variable,
determines whether the economy is in high or low inflation environment.
The general price inflation switches to high (low) inflation environment
when the exchange rate depreciates (appreciates). The result reveals that
the speed of adjustment (transiting between regimes) is 70%. This can be
considered to be relatively fast transition. This high speed can be
attributed to availability of information in the foreign exchange market
and the CBN announcement of any changes in exchange rate policy. The
transition in the inflation was found to be triggered whenever the naira
depreciates or appreciates by up to N75. This signifies the threshold value
of the naira that triggers the regime change in the general price level. This result bears some similarity with other developing African economies like Ghana, Kenya etc., while the shape of the Philips curve bears some similarities with that of Australia reported in Bohm (2001), also Kavkler and Bohm (2005).

Compared to the first regime, there is evidence of high persistent in inflation in the second regime. This means high exchange rate depreciation regime is characterized not only by higher inflation but also by higher persistent inflation. Furthermore, the coefficient of marginal cost is relatively higher in this regime. This suggests high exchange rate depreciation regime connotes an upward shift in the Philips curve, therefore the slope is steeper and more elastic for the period under review. Unlike the low exchange rate regime, supply side factor (cost push) are not the drivers of inflation. Therefore, it can be inferred that the monetary policy authority effort of achieving disinflation at a high exchange rate regime must take account of stabilizing exchange rate of the naira, particularly when inflation is in a high regime.

5.0 Conclusion and Recommendation

This paper examines inflation dynamics and exchange rate pass-through in a nonlinear NKPC model for Nigeria economy. There are two central messages from this paper. First, exchange rate is found to have substantial influence on the dynamics of inflation in Nigeria, with its level of depreciation as the trigger of the regime-switching behavior of inflation. Secondly, the nonlinear augmented NKPC is found to be empirically relevant for Nigeria, and the nature of inflation dynamics is unique across two different inflation regimes. High exchange rate depreciation regime, for instance, is analogous to high inflation regime and, as such, inflation is persistent. This means that achieving effective disinflation during such regime is possible when exchange rate is stabilized.

The empirical evidence provides support for an augmented nonlinear NKPC model because we prove that exchange rate has key and substantial influence in inflation dynamics in Nigeria. The regime switching behaviour of inflation in our open economy is largely explained by the exchange rate regime of the domestic currency as found in the transition function. Overall, the empirical relevance of a nonlinear augmented NKPC explains the uniqueness of persistence of inflation across different
regimes and the tendency of monetary authority to achieve disinflation across the two exchange rate regimes (having established a low inflation persistence of -0.43 in the low exchange rate depreciation regime and higher inflation persistence of 1.67 in the high exchange rate depreciation regime). Thus, high exchange rate depreciation regime is analogous to high inflation regime as such inflation is persistent. This means that achieving exchange rate stability is a necessary condition for an effective disinflation during high inflation regime. This paper also provides an empirical evidence of an upward shift in Philips curve during a high exchange rate depreciation regime. In addition, the empirical evidence points out the dominance of food inflation in the consumption basket. Therefore, this paper recommends that monetary policy response to low inflation regime must target the various components of the consumption basket, while efforts to curtail persistent inflation in high inflation regime must include exchange rate management strategy that keeps the relative value of the naira stable.

References


Inflation Dynamics and Exchange Rate Pass-Through in Nigeria: Evidence from Augmented Nonlinear New Keynesian Philips Curve


Lagoa, P. (2014), Open economy New Keynesian Philips Curve for Group of 7 countries


Appendix I:
Result of Augmented Philips Curve in Low Exchange rate Depreciation Regime (forward looking)

\[ \pi_t = (\beta_0 + \varphi_1^f E_t \pi_{t+1} + \varphi_2^b \Delta \pi_{t-1} + \varphi_3 \Delta \pi_e + \varphi_4 \Delta \pi_f + \vartheta \Delta \pi_{imp} + \beta \Delta rmc_t + \lambda \Delta exr_t) G(s_{t-1}, \gamma, \epsilon) + \epsilon_{\pi t} \]

<table>
<thead>
<tr>
<th>Linear Parameters</th>
<th>Coefficient</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 )</td>
<td>Restricted</td>
<td>-</td>
</tr>
<tr>
<td>( \varphi_1^f )</td>
<td>0.18233</td>
<td>0.0002</td>
</tr>
<tr>
<td>( \varphi_2^b )</td>
<td>0.03960</td>
<td>0.3488</td>
</tr>
<tr>
<td>( \varphi_3 )</td>
<td>0.36387</td>
<td>0.0000**</td>
</tr>
<tr>
<td>( \varphi_4 )</td>
<td>0.40165</td>
<td>0.0000**</td>
</tr>
<tr>
<td>( \vartheta )</td>
<td>0.12332</td>
<td>0.0000**</td>
</tr>
<tr>
<td>( \beta_{1,t} )</td>
<td>0.00076</td>
<td>0.9280</td>
</tr>
</tbody>
</table>
Appendix 2

Result of Augmented Philips Curve and Exchange Rate Pass-Through in High Inflation Regime

<table>
<thead>
<tr>
<th>Nonlinear variables</th>
<th>Parameter</th>
<th>Estimate</th>
<th>p-value</th>
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<td>$\varphi_{2,2}$</td>
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<tr>
<td></td>
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<tr>
<td></td>
<td>$\lambda_{2,t}$</td>
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<td>0.9998</td>
</tr>
<tr>
<td></td>
<td>$\varphi_{2,3}$</td>
<td>-0.36387</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>$\varphi_{2,4}$</td>
<td>-0.40165</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>$\theta_{2,2}$</td>
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<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
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</tr>
<tr>
<td></td>
<td>$c_1$</td>
<td>366.20022</td>
<td>0.2327</td>
</tr>
</tbody>
</table>

\[ G(s_{t-\gamma}, \gamma, c) = \left[ (1 + \exp[-\gamma(s_{t-\gamma}, c)]) - 2 \right] \]