Improving Accuracy with Forecast Combination: the Case of Inflation and Currency in Circulation in Nigeria

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This study shows how the application of forecast combination improves the accuracy of forecasts of economic variables. Using data from January 2009 to December 2014 on the Nigerian inflation rate, and forecasts of currency in circulation (see Ikoku, (2014)) as examples, we find that by combining forecasts of both variables using the regression-based method, the mean absolute percent errors of the combined forecasts were lower than the forecast errors from the individual models of the variables.

Keywords: Inflation forecasting, liquidity forecasting, forecast combination

JEL Classification: G12, G15

1.0 Introduction

It is known that forecast models are not always accurately specified and often lack properties required to track certain changes in the variable of interest, Hendry (2002). Following this, the call for a combination of the various forecasts to improve on the individual forecasts came to the fore, giving rise to our motivation to apply forecast combinations especially when the development of a more robust and properly specified model is not feasible.

There has been an increase in the number of studies on the use of forecast combinations to reduce forecast errors. Bjornland et,al. (2010) combined a number of Norges Bank inflation forecasts from a baseline AR model, ARIMA, variations of the VAR model and a number of other econometric techniques, among others, to produce new forecasts of inflation. The results of their study indicated that the combined forecast of the top 8 models and top 20 models had lower forecast errors

[1] Corresponding Author: The views expressed in this paper are those of the authors alone and do not necessarily represent those of the Central Bank of Nigeria or CBN policy. We would like to thank several anonymous reviewers for their comments on earlier drafts of this paper.

compared with the benchmark AR(4) model. In addition, the variance between the forecast and the actual variable appeared to decrease as the forecast horizon increased beyond the 1-step ahead forecast. Relating the combination of individual forecasts to the portfolio diversification process, which results in lower variances, Diebold (2006) explains that by combining forecasts, the forecast errors of the combined forecast will also have a lower variance, indicating that it is more unbiased.

This paper proposes to determine whether the accuracy of Nigerian liquidity and inflation rate forecasts could be improved by combining the forecasts of various econometric models to produce a more optimal forecast with lower forecast errors than the individual forecasts. Our motivation for this study is driven by the need to improve on the forecast accuracy of economic variables, especially inflation and currency in circulation, in line with the Central Bank of Nigeria's monetary policy objectives.

2.0 Literature Review

The ground breaking study on the combination of forecasts by Bates and Granger (1969) explained that if the goal of a forecasting exercise is to produce the best possible forecast of a variable, then selecting one out of two forecasts, based on its forecast error would not be the best option. This is because both forecast models might contain uniquely independent information that are mutually exclusive and could be combined to obtain a more accurate forecast. According to the authors, this independent information could include assumptions about the relationships between variables in the forecast models. Granger and Bates (1969), however, require that for forecasts to be combined, the individual forecasts should be unbiased. They explain that by combining a biased and an unbiased forecast, the combined forecast becomes biased.

Following further research, Diebold (1989) echoed the views of Ericsson (1989) that individual forecasts pooled together to obtain a newer and more accurate forecast (by measure of a root mean square error), are products of mis-specified econometric models. Ericson explains that such mis-specified models could be improved to produce correctly specified models with equally lower forecast errors similar to the pooled forecast. Diebold goes on to explain that the technique of forecast combination can be applied appropriately when there exists constraints preventing the development of a correctly specified model, by combining all information contained in the individual models, rather than their forecasts. Diebold and Lopez (1996) assert that the failure of a model's forecast to encompass another is evidence that both models are mis-specified, hence, producing a regression based forecast combination that has lower forecast errors. The authors echo the inferences of Diebold (1989) in advocating for a combination of information sets, as opposed to the combination of forecasts. Diebold and Lopez also state that combining forecasts could be referred to as the connection between real-time forecasting, in the short run, and the long-run process of developing a more robust and properly specified model.

In a study on univariate time series forecasting, Newbold and Granger (1974) indicate their preference for the use of Holt-Winters exponential smoothening prediction technique if the number of observations are less than 30. They, however, propose stepwise autoregressive forecasts in situations where the number of observations exceeds 30. Noting situations where the observations lie between 40 and 50, they propose the combination of forecasts from the Holt-Winters simulation and the stepwise autoregressive method. The authors also advocate the use of the Box-Jenkins methodology when the observations are between 40 and 50 and a possible combination of the forecasts from the Box-Jenkins, Holt-Winters and the stepwise auto-regression. Citing data sets with strong seasonal trends and extreme volatility, Newbold and Granger suggested the use of the Harrison method in place of the Holt-Winters for optimal forecast results.

In recent times, the application of forecast combination to improve the accuracy of predicting various economic variables has become more popular among studies on developed economies. Ekland and Karlsson (2005) combined a number of Swedish inflation forecasts by determining the weights based on the predictive likelihood, deviating from the standard procedure of the marginal likelihood in a bid to avoid in-sample over fitting. Adopting a Bayesian approach, the authors made use of quarterly data from 1983Q1 to 2003Q4 and discovered that the predictive likelihood method reduced the forecast error (RMSE) by 37 percent, compared with the marginal likelihood approach.

Implementing a combination approach to improve GDP estimates, Aruoba et.al. (2011) obtained estimates of GDP based on the level of income and expenditure. Highlighting the strength of forecast combinations to bring about the optimal harnessing of information contained in the two estimates, they used quarterly data from 1947Q2 – 2009Q3 to show that by combining forecasts, using Bayesian techniques, the accuracy of GDP estimates could be improved.

Taking note of a typical environment where individual models are susceptible to structural breaks and misspecification, Samuels and Sekkel (2013) opined that selecting any particular model as the best was not as reliable as pooling the forecasts from the varying models estimated. They further stated that, by taking the arithmetic average of the different forecasts, the combined forecast did outperform forecasts made from combinations that required the use of sophisticated estimation techniques to determine the weight assigned to the individual forecasts.

Applying the forecast combination technique to the prediction of hedge funds returns, Panopoulou and Vrontos (2015) implemented a simple averaging method in addition to other more complex methodologies. They opined that the simple forecast combination techniques outperform the other methodologies, which involved the combination of information sets. Their results indicated that by dynamically constructing portfolios based on the combination forecasts of hedge funds returns led to improved portfolio performance forecasts.

Cheng and Yang (2015) studied the impact of loss functions on the combining weights used in the combination of forecasts, indicating the key role loss function plays as a key ingredient in formulating combinations, and in their use for the definition of performance evaluation criteria. Concluding their study, the authors noted that quadratic loss has been most widely used; however, they opined that this might result in combined forecasts that may be subject to undue influence from few forecast outliers. Despite the robustness of absolute loss, the authors assert the likelihood of producing more outlier forecasts than the quadratic loss.

Allowing the combining weights to be dependent on the variable being forecasted, Kapetanios et.al. (2015) cites the example of where the forecast density of the variable of interest is realized (for example, the state of the economy). They opine that while one model may outperform another model in a recession, the other model might perform better in a bullish market. Considering piecewise linear weight functions due to their ability to explicitly allow the combining weights to vary based on the specific regions, among others, the authors showed that by ranking the models based on their ability to forecast under a certain region, their generalized combinations works better in practice.

From our review, little progress in the application of forecast combinations on underdeveloped nations has been made. Andrawis et.al. (2010) developed a combined forecast of Egyptian tourism demand in one of the few studies in an emerging economy. They adopt a modified method by combining short and long-term forecasts [for example, combining a 36-month forecast (monthly data) with a 3-year forecast (annual data) of the same variable]. Applying a variety of methods and set of assumptions to determine the optimal weights for the combined forecast, the authors show that the combined forecast did outperform those from the individual forecast models.

3.0 Data and Methodology

3.1 Data

Data used for this study include monthly inflation rates (referred to hereafter, as CPI for convenience in this study) and forecasts of currency in circulation (CIC) from Ikoku (2014). Also, the monetary policy rate (MPR) and interbank exchange rate (EXR) used in this study as explanatory variables in the estimation of inflation rate were sourced from the statistics database of the CBN. Making use of the rebased inflation rate data on the Nigerian economy, we obtained data from the National Bureau of Statistics ranging from January 2009 to December 2014, making for 72 observations.

3.2 Methodology

In line with the objective of this paper, we develop three forecast models for forecasting the inflation rate, namely an AR(1) model, an optimized ARIMA model and a structural-ARIMA (SARIMA) model of year-on-year inflation rates. Data from January 2009 to December 2013 was used as the estimation sample, while testing the accuracy of the forecast using out of sample data from January 2014 to December 2014. All possible combinations of forecasts from the three models were produced to determine the impact of the resulting combinations on the forecast accuracy of the variables. We also combine monthly forecasts from Nigerian currency in circulation (CIC) models developed by Ikoku (2014) to determine if there is a decrease in the mean absolute percent error (MAPE) following the combination of forecasts.

Following Diebold (2006), we combine forecasts from the inflation models, we further apply the same method to the CIC forecasts. In both instances, we use the regression approach shown in the equation below:

$$y_{t+h} = \beta_a y^a_{t+h,t} + \beta_b y^s_{t+h,t} + \varepsilon_{t+h,t} \quad (1)$$

Where, for example, y_{t+h} is the time series (actual) of the desired variable (CPI), $y^a_{t+h,t}$ is the forecast produced from the ARIMA model, $y^s_{t+h,t}$ is the forecast produced from the Structural-ARIMA (SARIMA) model, and β_a is the coefficient of the relationship between the actual inflation rate data and the ARIMA forecast. β_b is the coefficient of the relationship between the actual inflation data and the SARIMA forecast, while $\varepsilon_{t+h,t}$ is the error term of the regression. The two coefficients (combining weights) sum up to unity and the intercept is ignored. We replicate this procedure when combining the forecasts of the CIC. This results in 3 possible combinations of the CPI forecasts and 6 combinations of the CIC forecasts. The CPI forecast combinations would include; ARIMA and SARIMA, AR(1) and ARIMA, and AR(1) and SARIMA, while the CIC combinations include; AR(1) and ARIMA, AR(1) and SARIMA, AR(1) and VECM, ARIMA and SARIMA, ARIMA and VECM, and lastly SARIMA and VECM.

4.0 Empirical Results

4.1 Descriptive Statistics

Presented in Figures 1, 2 and 3 are charts of the CPI, MPR and exchange rate over the sample period. Also presented in Figures 4, 5 and 6 are descriptions of the statistics of the CPI, MPR and EXR, respectively. The inflation rate recorded a maximum of 15.65 percent in February of 2010, with an average of 11.00% over the sample period. The general price level was its lowest as the CPI was 7.71 percent in February 2014. At the 5% significance level, the Jarque-Bera test revealed that CPI was normal with a statistically significant probability value of 0.09.

The MPR averaged 9.76 percent under the sample period, peaking at 13% from November 2014 to December 2014. The benchmark rate had a low of 6% between July 2009 and August 2010. The test of normality was rejected for MPR with a Jarque-Bera of 9.89 and a statistically significant probability value of 0.007 at the 5% significance level. Over the sample period, exchange rates averaged N154.31 per dollar, recording a maximum of N167.50 per dollar in December 2014 and a minimum of N145.95 per dollar over the sample period. The distribution of exchange rates indicates that it is normally distributed at the 5% significance level, with a Jarque-Bera of 0.69 and a prob value of 0.71.

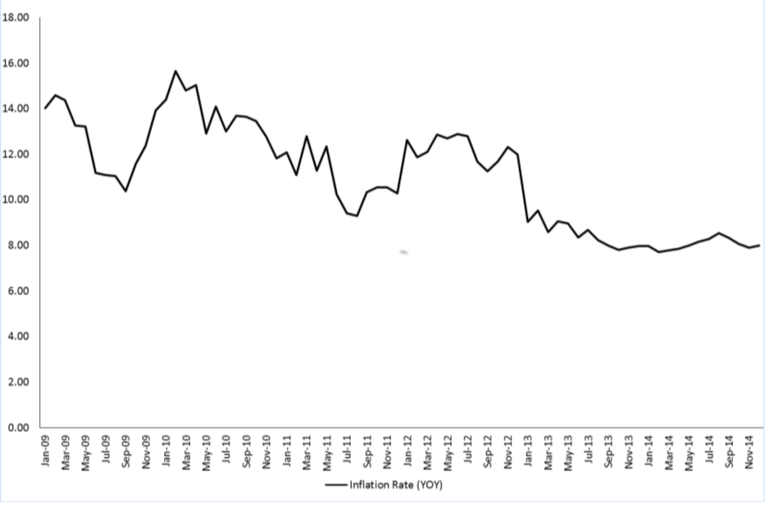

Figure 1: Inflation rate (YoY) (January 2009 – December 2014)

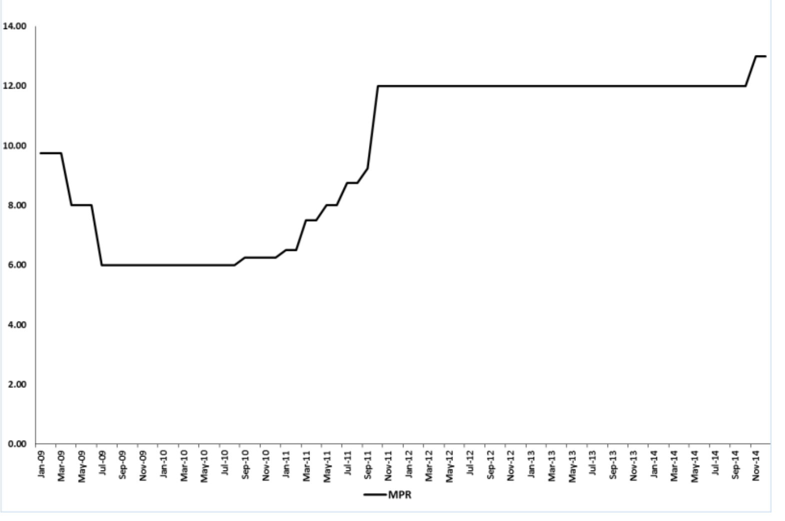

Figure 2: Graph of monetary policy rate (January 2009 – December 2014)

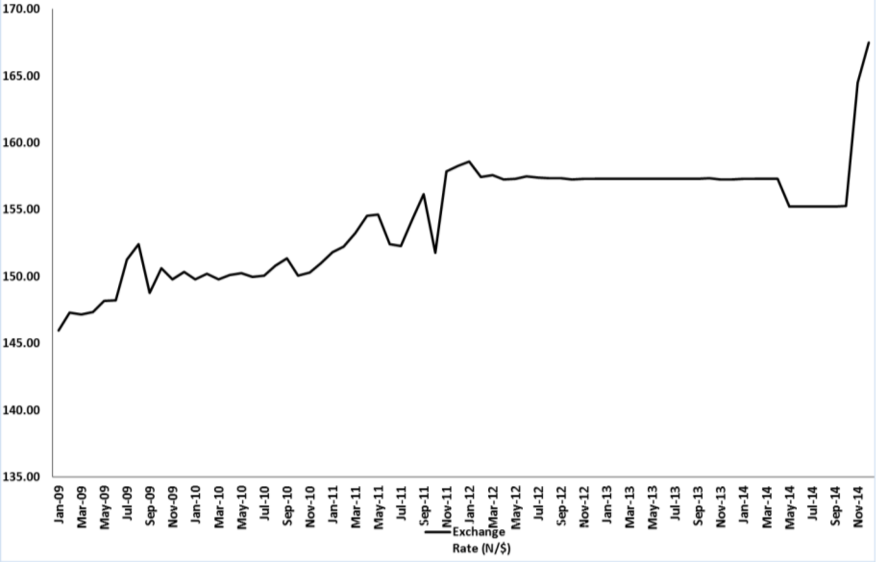

Figure 3: Graph of exchange rate (Naira/Dollar) (January 2009 – December 2014)

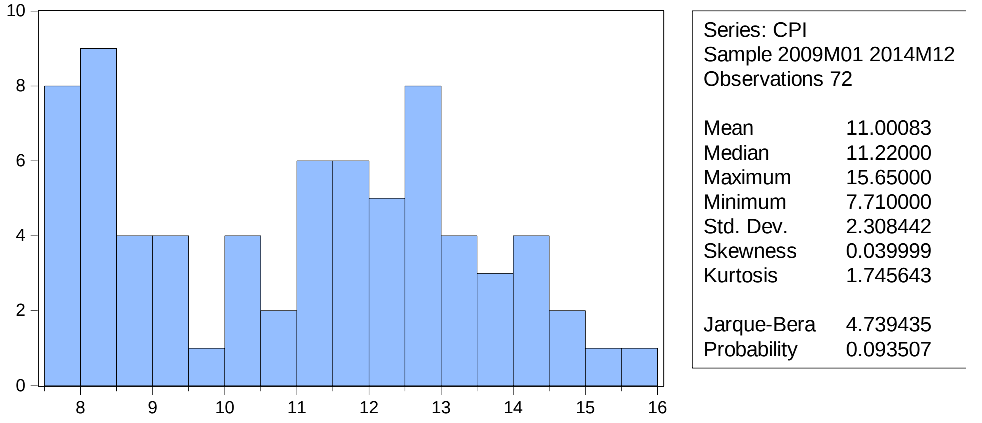

Figure 4: Descriptive Statistics (Inflation)

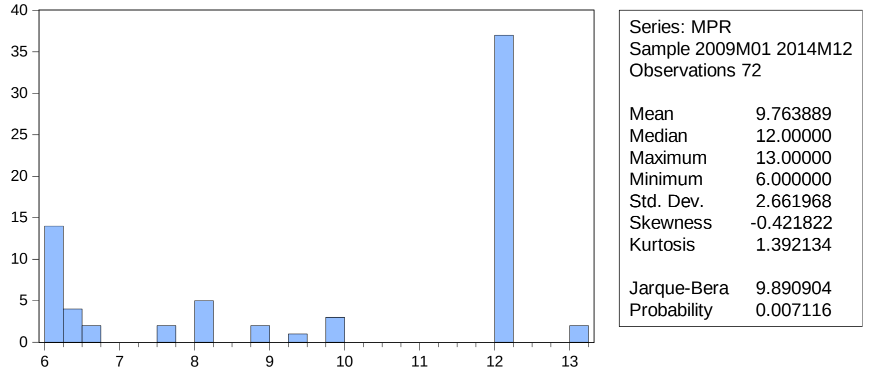

Figure 5: Descriptive Statistics (Monetary Policy Rate)

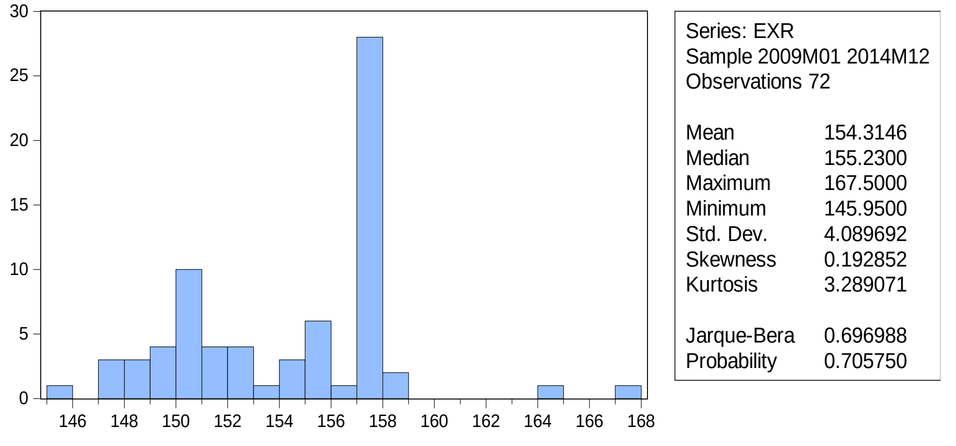

Figure 6: Descriptive Statistics (Exchange Rate)

4.2 Unit Root Tests

Regression models built with non-stationary variables, i.e, variables containing unit roots, have been shown to produce misleading coefficients (due to a downward bias in the estimation of least squares). Following this finding, we apply the test proposed by Dickey and Fuller (1976) for the presence of unit roots in all the variables used in the study. In addition to applying the Augmented Dickey-Fuller (ADF) test for stationarity, we also test using the Phillips-Perron (PP) (1998) test, which accounts for possible autocorrelation and heteroskedasticity in the error terms of the regression model. Following Bierens (2003) in which an autoregressive process (equation (2)), is transformed by recursively replacing the autoregressive terms with differenced terms of the variable, we get the result in equation 3 below.

$$y_t = \beta_0 + \sum_{j=1}^{k} \beta_j y_{t-j} + U_t \qquad (2)$$
$$U_t \sim iidN(0, \sigma^2)$$

$$\Delta y_t = \alpha_0 + \sum_{j=1}^{k} \alpha_j \Delta y_{t-j} + \alpha_k y_{t-k} + U_t \qquad (3)$$
$$U_t \sim iidN(0, \sigma^2)$$

where $\alpha_0 = \beta_0, \alpha_j = \sum_{j=1}^{k} \beta_j - 1,\ j = 1, \ldots, k.$

We apply this test for stationarity on the inflation rate, exchange rate and MPR with the condition that the variable has a unit root when $\alpha_j = 0$ as the null hypothesis (H_0), against an alternative hypothesis (H_1) when $\alpha_k < 0$, which indicates the absence of a unit root. The results of the stationarity test using both the ADF and PP tests as shown in Table 1 reached similar conclusions. On the levels, CPI, EXR and MPR were non-stationary, with assumptions of trend and without. Following the differencing of the variables, both tests revealed that all variables were stationary.

Table 1: Unit root tests

Augmented Dickey Fuller					
Null Hypothesis: Variable has a unit root			Null Hypothesis: Variable has a unit root		
	Levels		**First Difference**		
	McKinnon Prob-Values without Trend	McKinnon Prob-Values with Trend	McKinnon Prob-Values without Trend	McKinnon Prob-Values with Trend	**Test Results**
CPI	0.5635	0.0989	0.0001	0.0000	I(1)
Exchange Rate	0.7866	0.2438	0.0000	0.0000	I(1)
Monetary Policy Rate	0.9168	0.3168	0.0000	0.0000	I(1)
Philips Perron					
Null Hypothesis: Variable has a unit root			Null Hypothesis: Variable has a unit root		
	Levels		**First Difference**		
	Prob-Values without Trend	Prob-Values with Trend	Prob-Values without Trend	Prob-Values with Trend	**Test Results**
CPI	0.5349	0.2256	0.0001	0.0000	I(1)
Exchange Rate	0.8232	0.2438	0.0000	0.0000	I(1)
Monetary Policy Rate	0.8568	0.2862	0.0000	0.0000	I(1)

4.3 Interpretation of Results

4.4 Inflation Models

In line with the objective of showing how forecast combination improves the forecast accuracy of an economic variable, we built three different inflation models as shown in Table 2. The AR(1) model had an adjusted R-Squared of 0.0428, which means that the model was only able to explain approximately 4.28 percent of the variations in the inflation rate. The ARIMA (3, 1, 1) recorded an adjusted R-Squared of 0.1845, which implied that the autoregressive and moving average terms were able to explain 18.45 percent of the variations in the inflation rate. Estimated in a sample range of January 2009 to December 2013, the correlogram of both models showed that the error terms were white noise. Producing a 12-month out of sample forecast of inflation from January to December 2014 as shown in Table 3, the AR(1) and ARIMA model recorded a Mean Absolute Percent Error of 10.48 and 3.98 percent, respectively.

We also built a structural-ARIMA model with the inclusion of the MPR and EXR as explanatory variables. The model recorded an improvement in the adjusted R-Squared from the ARIMA model with a value of 0.6874, while the error terms were white noise as noted in the correlogram. The results of the forecast from this model, applying the same forecast sample specification as those followed in producing the forecast from the AR(1) and ARIMA models, recorded an MAPE of 16.49 percent.

Table 2: Inflation forecast models

Inflation Forecast Models						
	AR(1)		ARIMA		Structural-ARIMA	
	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
D(MPR(-2))	-	-	-	-	-0.7107	0.0020
D(EXR(-1))	-	-	-	-	0.3362	0.0000
@SEAS(2)	-	-	-	-	-0.4585	0.0150
@SEAS(11)	-	-	-	-	0.8683	0.0000
@SEAS(12)	-	-	-	-	-0.2843	0.0392
AR(1)	-0.2432	0.0648	0.7496	0.0000	-1.1085	0.0000
AR(2)	-	-	0.4204	0.0089	-0.4843	0.0011
AR(3)	-	-	-0.4046	0.0034	-	-
MA(1)	-	-	-0.9999	0.0000	1.0109	0.0000
MA(2)	-	-	-	-	0.9938	0.0000
SMA(12)	-	-	-	-	-0.9436	0.0000
C	-0.1125	-0.2432	-0.0928	0.0000	-0.1032	0.1087
Adj R-squared	0.0428		0.1845		0.6874	
AIC	2.7834		2.6915		1.8428	
SIC	2.8544		2.8723		2.2442	

Table 3: Inflation forecast model performance

Inflation Forecast Model Performance	
Model Estimation Sample	January 2009 - December 2013 (60 Months)
Forecast Sample	January 2014 - December 2014 (12 Months)
Inflation Forecasts	
AR(1)	
Root Mean Squared Error	1.0030
Mean Abs. Percent Error	10.4785
ARIMA	
Root Mean Squared Error	0.3564
Mean Abs. Percent Error	3.9759
Structural-ARIMA	
Root Mean Squared Error	1.4524
Mean Abs. Percent Error	16.4879

Table 4: Correlogram for AR(1) model

Sample: 2009M01 2013M12
Included observations: 58
Q-statistic probabilities adjusted for 1 ARMA term

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
. | . |	. | . |	1	0.061	0.061	0.2245	
. |** |	. |** |	2	0.227	0.224	3.4196	0.064
.*| . |	.*| . |	3	-0.162	-0.197	5.0829	0.079
. | . |	. | . |	4	-0.009	-0.039	5.0886	0.165
.*| . |	. | . |	5	-0.077	0.010	5.4763	0.242
.*| . |	.*| . |	6	-0.087	-0.112	5.9818	0.308
.*| . |	.*| . |	7	-0.099	-0.083	6.6440	0.355
. | . |	. | . |	8	-0.056	-0.010	6.8590	0.444
.*| . |	.*| . |	9	-0.080	-0.079	7.3146	0.503
.*| . |	.*| . |	10	-0.119	-0.144	8.3360	0.501
. | . |	. | . |	11	-0.009	0.026	8.3414	0.596
**| . |	**| . |	12	-0.288	-0.318	14.619	0.201
. |*. |	. |*. |	13	0.177	0.181	17.032	0.148
.*| . |	. | . |	14	-0.080	-0.014	17.543	0.176
. | . |	.*| . |	15	0.061	-0.200	17.844	0.214
.*| . |	.*| . |	16	-0.193	-0.182	20.938	0.139
. | . |	.*| . |	17	-0.057	-0.076	21.212	0.171
. | . |	.*| . |	18	-0.050	-0.094	21.431	0.208
. |*. |	. |*. |	19	0.139	0.081	23.144	0.185
.*| . |	**| . |	20	-0.096	-0.224	23.982	0.197
. | . |	.*| . |	21	0.062	-0.133	24.339	0.228
. | . |	. | . |	22	-0.015	-0.012	24.361	0.276
. |*. |	. | . |	23	0.077	-0.047	24.957	0.299
. |*. |	.*| . |	24	0.085	-0.137	25.689	0.316

Table 5: Correlogram for ARIMA model

Sample: 2009M01 2013M12
Included observations: 56
Q-statistic probabilities adjusted for 4 ARMA terms

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>. | . |</td><td>. | . |</td><td>1</td><td>-0.042</td><td>-0.042</td><td>0.1022</td><td></td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>2</td><td>-0.099</td><td>-0.101</td><td>0.6912</td><td></td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>3</td><td>-0.066</td><td>-0.076</td><td>0.9621</td><td></td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>4</td><td>0.009</td><td>-0.008</td><td>0.9673</td><td></td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>5</td><td>0.069</td><td>0.056</td><td>1.2735</td><td>0.259</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>6</td><td>-0.041</td><td>-0.040</td><td>1.3838</td><td>0.501</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>7</td><td>-0.027</td><td>-0.018</td><td>1.4309</td><td>0.698</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>8</td><td>0.053</td><td>0.053</td><td>1.6233</td><td>0.805</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>9</td><td>-0.086</td><td>-0.093</td><td>2.1356</td><td>0.830</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>10</td><td>0.017</td><td>0.012</td><td>2.1552</td><td>0.905</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>11</td><td>-0.035</td><td>-0.040</td><td>2.2450</td><td>0.945</td></tr>
<tr><td>***| . |</td><td>***| . |</td><td>12</td><td>-0.366</td><td>-0.392</td><td>12.111</td><td>0.146</td></tr>
<tr><td>. |*. |</td><td>. |*. |</td><td>13</td><td>0.134</td><td>0.100</td><td>13.475</td><td>0.142</td></tr>
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<tr><td>. | . |</td><td>. | . |</td><td>15</td><td>-0.002</td><td>-0.065</td><td>13.556</td><td>0.259</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>16</td><td>-0.157</td><td>-0.154</td><td>15.562</td><td>0.212</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>17</td><td>-0.148</td><td>-0.149</td><td>17.381</td><td>0.182</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>18</td><td>0.039</td><td>-0.088</td><td>17.511</td><td>0.230</td></tr>
<tr><td>. |*. |</td><td>. |*. |</td><td>19</td><td>0.150</td><td>0.118</td><td>19.493</td><td>0.192</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>20</td><td>-0.136</td><td>-0.169</td><td>21.165</td><td>0.172</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>21</td><td>0.044</td><td>-0.042</td><td>21.345</td><td>0.211</td></tr>
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<tr><td>. | . |</td><td>. | . |</td><td>23</td><td>0.043</td><td>-0.048</td><td>21.529</td><td>0.308</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>24</td><td>0.026</td><td>-0.148</td><td>21.599</td><td>0.363</td></tr>
</table>

Table 6: Correlogram for structural-ARIMA model

Sample: 2009M01 2013M12
Included observations: 55
Q-statistic probabilities adjusted for 5 ARMA terms and 5 dynamic regressors

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob*</th></tr>
<tr><td>. | . |</td><td>. | . |</td><td>1</td><td>0.038</td><td>0.038</td><td>0.0848</td><td></td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>2</td><td>0.028</td><td>0.026</td><td>0.1301</td><td></td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>3</td><td>-0.118</td><td>-0.120</td><td>0.9661</td><td></td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>4</td><td>-0.096</td><td>-0.089</td><td>1.5356</td><td></td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>5</td><td>0.021</td><td>0.036</td><td>1.5642</td><td></td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>6</td><td>-0.015</td><td>-0.026</td><td>1.5795</td><td>0.209</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>7</td><td>0.072</td><td>0.051</td><td>1.9187</td><td>0.383</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>8</td><td>-0.042</td><td>-0.048</td><td>2.0372</td><td>0.565</td></tr>
<tr><td>**| . |</td><td>**| . |</td><td>9</td><td>-0.291</td><td>-0.300</td><td>7.7933</td><td>0.099</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>10</td><td>-0.064</td><td>-0.039</td><td>8.0820</td><td>0.152</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>11</td><td>-0.044</td><td>-0.018</td><td>8.2172</td><td>0.223</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>12</td><td>0.047</td><td>-0.032</td><td>8.3771</td><td>0.301</td></tr>
<tr><td>. |*. |</td><td>. |*. |</td><td>13</td><td>0.194</td><td>0.158</td><td>11.201</td><td>0.191</td></tr>
<tr><td>. |*. |</td><td>. |*. |</td><td>14</td><td>0.140</td><td>0.138</td><td>12.700</td><td>0.177</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>15</td><td>-0.038</td><td>-0.087</td><td>12.815</td><td>0.234</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>16</td><td>-0.133</td><td>-0.083</td><td>14.228</td><td>0.221</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>17</td><td>-0.165</td><td>-0.151</td><td>16.467</td><td>0.171</td></tr>
<tr><td>.*| . |</td><td>**| . |</td><td>18</td><td>-0.124</td><td>-0.252</td><td>17.764</td><td>0.167</td></tr>
<tr><td>. |*. |</td><td>. |*. |</td><td>19</td><td>0.134</td><td>0.124</td><td>19.326</td><td>0.153</td></tr>
<tr><td>.*| . |</td><td>**| . |</td><td>20</td><td>-0.143</td><td>-0.237</td><td>21.149</td><td>0.132</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>21</td><td>-0.027</td><td>-0.117</td><td>21.215</td><td>0.170</td></tr>
<tr><td>**| . |</td><td>.*| . |</td><td>22</td><td>-0.266</td><td>-0.166</td><td>27.920</td><td>0.046</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>23</td><td>-0.015</td><td>0.032</td><td>27.941</td><td>0.063</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>24</td><td>-0.012</td><td>-0.102</td><td>27.955</td><td>0.084</td></tr>
</table>

Further to this, we combine the forecasts of all three models following equation 1 (see Table 7). The combination of the AR(1) and ARIMA model forecasts recorded a MAPE of 3.76 percent with weights of -0.1219 and 1.1212 to the AR(1) and ARIMA forecasts, respectively. This is significantly lower than the forecast errors of the individual AR(1) model, which was 10.48 percent, and slightly lower than the ARIMA model's MAPE of 3.98 percent. Combining the AR(1) and SARIMA forecasts with weights of 0.95 and 0.05, respectively, the resulting forecast recorded a forecast with MAPE of 10.60 percent, which was marginally higher than the forecast error of the AR(1) model of 10.48 percent, but lower than the SARIMA forecast error of 16.49 percent. The third combination, the ARIMA and the SARIMA model forecasts, recorded an MAPE of 3.95 percent, which was lower than both the ARIMA and SARIMA models forecast errors. (See Tables 7 and 8).

Table 7: Forecast combination regressions

Forecast Combination Regressions Results				
	Inflation Combination		CIC Combination	
	Coefficient	Prob.	Coefficient	Prob.
Combination 1				
AR(1)	-0.1219	0.0182	-0.8209	0.0000
ARIMA	1.1212	0.0000	1.8192	0.0000
Combination 2				
AR(1)	0.9482	0.0000	-0.1380	0.0000
SARIMA	0.0599	0.5725	1.1408	0.0000
Combination 3				
ARIMA	0.9572	0.0000	0.0194	0.6521
SARIMA	0.0436	0.1393	0.9845	0.0000
Combination 4				
AR(1)	-	-	-1.7829	0.0000
VECM	-	-	2.7882	0.0000
Combination 5				
ARIMA	-	-	0.9103	0.0000
VECM	-	-	0.0942	0.5539
Combination 6				
SARIMA	-	-	1.2532	0.0000

Table 8 – Inflation forecast combination results

Inflation Forecast Combination Result					
	ARIMA FORECAST	SARIMA FORECAST	COMBINATION BY REGRESSION	ACTUAL	ABSOLUTE PERCENT ERROR
Jan-14	8.11	7.80	8.09	7.98	0.0143
Feb-14	8.18	6.93	8.13	7.71	0.0544
Mar-14	8.25	6.85	8.19	7.78	0.0531
Apr-14	8.25	6.59	8.19	7.85	0.0427
May-14	8.23	6.69	8.17	8.00	0.0212
Jun-14	8.17	6.38	8.10	8.17	0.0091
Jul-14	8.09	6.30	8.02	8.28	0.0319
Aug-14	7.99	6.56	7.93	8.53	0.0703
Sep-14	7.88	6.58	7.83	8.32	0.0588
Oct-14	7.77	6.30	7.71	8.06	0.0429
Nov-14	7.67	7.28	7.65	7.90	0.0316
Dec-14	7.56	9.72	7.65	8.00	0.0442
				MAPE	0.0395
	AR(1) FORECAST	ARIMA FORECAST	COMBINATION BY REGRESSION	ACTUAL	ABSOLUTE PERCENT ERROR
Jan-14	7.81	8.11	8.14	7.98	0.0204
Feb-14	7.70	8.18	8.24	7.71	0.0683
Mar-14	7.59	8.25	8.33	7.78	0.0705
Apr-14	7.48	8.25	8.34	7.85	0.0630
May-14	7.36	8.23	8.34	8.00	0.0419
Jun-14	7.25	8.17	8.28	8.17	0.0131
Jul-14	7.14	8.09	8.20	8.28	0.0095
Aug-14	7.03	7.99	8.10	8.53	0.0501
Sep-14	6.91	7.88	8.00	8.32	0.0385
Oct-14	6.80	7.77	7.89	8.06	0.0211
Nov-14	6.69	7.67	7.78	7.90	0.0148
Dec-14	6.58	7.56	7.68	8.00	0.0403
				MAPE	0.0376
	AR(1) FORECAST	SARIMA FORECAST	COMBINATION BY REGRESSION	ACTUAL	ABSOLUTE PERCENT ERROR
Jan-14	7.81	7.80	7.81	7.98	0.0219
Feb-14	7.70	6.93	7.66	7.71	0.0059
Mar-14	7.59	6.85	7.55	7.78	0.0294
Apr-14	7.48	6.59	7.43	7.85	0.0532
May-14	7.36	6.69	7.33	8.00	0.0838
Jun-14	7.25	6.38	7.21	8.17	0.1178
Jul-14	7.14	6.30	7.10	8.28	0.1429
Aug-14	7.03	6.56	7.00	8.53	0.1790
Sep-14	6.91	6.58	6.90	8.32	0.1710
Oct-14	6.80	6.30	6.78	8.06	0.1593
Nov-14	6.69	7.28	6.72	7.90	0.1496
Dec-14	6.58	9.72	6.73	8.00	0.1583
				MAPE	0.1060

4.5 Currency in Circulation Models

Testing this methodology on the monthly forecasts produced in Ikoku (2014), the monthly forecast models of CIC included an AR(1) model, an ARIMA model, a structural-ARIMA (SARIMA) model and a Vector Error Correction model (VECM). Applying an estimation sample of between January 2000 and June 2010, a forecast sample of July 2010 to December, 2010, the AR(1) model, the baseline model, recorded an MAPE of 7.74 percent, while the ARIMA and the structural-ARIMA model recorded MAPE of 5.23 and 3.00 percent, respectively. Finally, the VECM forecast had a MAPE of 3.29 percent. In line with the goal of determining the effectiveness of using forecast combinations, we apply the same combination techniques in combining the AR(1) and ARIMA models' forecast with weights of -0.82 and 1.82, respectively, which yielded a combined forecast with a MAPE of 5.18 percent. This was noted to be lower than the individual forecast errors recorded in the models. The five other forecast combinations computed include the combinations of AR(1) and SARIMA, AR(1) and VECM, ARIMA and SARIMA, ARIMA and VECM, and SARIMA and VECM. These combinations had mean absolute percent errors of 2.69, 5.52, 2.99, 5.23 and 2.53 percent, respectively. The errors were all noted to be lower than the individual forecasts, which make up the combinations (See Tables 9a and 9b).

Table 9a – Currency in circulation forecast model performance

Currency in Circulation forecast model performance	
Model Estimation Sample	January 2000 - June 2010 (126 Months)
Forecast Sample	July 2010 - December 2010 (6 Months)
CIC Forecasts	
AR(1)	
Root Mean Squared Erro	137.4017
Mean Abs. Percent Error	7.7439
ARIMA	
Root Mean Squared Erro	87.3923
Mean Abs. Percent Error	5.234
Structural-ARIMA	
Root Mean Squared Erro	40.4139
Mean Abs. Percent Error	3.0037
VECM	
Root Mean Squared Erro	41.6138
Mean Abs. Percent Error	3.2953
Theil Inequality Coeffici	0.0201

Table 9b: Currency in circulation forecast combination result

Currency in Circulation Forecast Combination Result					
	AR(1) FORECAST	ARIMA FORECAST	COMBINATION BY REGRESSION	ACTUAL	ABSOLUTE PERCENT ERROR
Jul-10	1067.74	1121.80	1,166.14	1,076.92	0.0828
Aug-10	1071.81	1129.24	1,176.33	1,094.71	0.0746
Sep-10	1075.87	1136.31	1,185.87	1,125.39	0.0537
Oct-10	1079.90	1118.55	1,150.24	1,153.17	0.0025
Nov-10	1083.90	1146.91	1,198.58	1,227.64	0.0237
Dec-10	1087.88	1191.52	1,276.51	1,378.02	0.0737
				MAPE	0.0518
	AR(1) FORECAST	SARIMA FORECAST	COMBINATION BY REGRESSION	ACTUAL	ABSOLUTE PERCENT ERROR
Jul-10	1067.74	1068.15	1,068.21	1076.92	0.0081
Aug-10	1071.81	1076.50	1,077.16	1094.71	0.0160
Sep-10	1075.87	1072.62	1,072.17	1125.39	0.0473
Oct-10	1079.90	1104.06	1,107.45	1153.17	0.0397
Nov-10	1083.90	1167.05	1,178.70	1227.64	0.0399
Dec-10	1087.88	1355.13	1,392.54	1378.02	0.0105
				MAPE	0.0269
	AR(1) FORECAST	VECM FORECAST	COMBINATION BY REGRESSION	ACTUAL	ABSOLUTE PERCENT ERROR
Jul-10	1067.74	1058.49	1,041.95	1076.92	0.0325
Aug-10	1071.81	1065.31	1,053.67	1094.71	0.0375
Sep-10	1075.87	1066.59	1,049.98	1125.39	0.0670
Oct-10	1079.90	1057.05	1,016.15	1153.17	0.1188
Nov-10	1083.90	1113.36	1,166.10	1227.64	0.0501
Dec-10	1087.88	1179.45	1,343.37	1378.02	0.0252
				MAPE	0.0552
	ARIMA FORECAST	SARIMA FORECAST	COMBINATION BY REGRESSION	ACTUAL	ABSOLUTE PERCENT ERROR
Jul-10	1121.80	1068.15	1,069.22	1076.92	0.0071
Aug-10	1129.24	1076.50	1,077.56	1094.71	0.0157
Sep-10	1136.31	1072.62	1,073.90	1125.39	0.0458
Oct-10	1118.55	1104.06	1,104.35	1153.17	0.0423
Nov-10	1146.91	1167.05	1,166.65	1227.64	0.0497
Dec-10	1191.52	1355.13	1,351.85	1378.02	0.0190
				MAPE	0.0299
	ARIMA FORECAST	VECM FORECAST	COMBINATION BY REGRESSION	ACTUAL	ABSOLUTE PERCENT ERROR
Jul-10	1121.80	1058.49	1,121.80	1076.92	0.0417
Aug-10	1129.24	1065.31	1,129.24	1094.71	0.0315
Sep-10	1136.31	1066.59	1,136.31	1125.39	0.0097
Oct-10	1118.55	1057.05	1,118.55	1153.17	0.0300
Nov-10	1146.91	1113.36	1,146.91	1227.64	0.0658
Dec-10	1191.52	1179.45	1,191.52	1378.02	0.1353
				MAPE	0.0523
	SARIMA FORECAST	VECM FORECAST	COMBINATION BY REGRESSION	ACTUAL	ABSOLUTE PERCENT ERROR
Jul-10	1068.15	1058.49	1,070.56	1076.92	0.0059
Aug-10	1076.50	1065.31	1,079.30	1094.71	0.0141
Sep-10	1072.62	1066.59	1,074.13	1125.39	0.0456
Oct-10	1104.06	1057.05	1,115.82	1153.17	0.0324
Nov-10	1167.05	1113.36	1,180.48	1227.64	0.0384
Dec-10	1355.13	1179.45	1,399.05	1378.02	0.0153
				MAPE	0.0253

5.0 Conclusion

While the accurate prediction of an economic variable remains extremely difficult, the techniques shown in this paper support the claim that by combining forecasts using the appropriate methodology, policymakers could improve the accuracy of their projections. Hence, this would improve the quality of policies emanating from them. Having said this, it would be more beneficial to develop a better-specified model, which combines all information sets contained in the different forecasts combined. While time constraints preclude this best-case scenario, we would like to suggest further research in this area to improve the prediction of target economic variables.

References

Andrawis, R. R., Atiya, A. F. & El-Shishiny. H. (2010). Combination of long term and short-term forecasts, with application to tourism demand forecasting. *International Journal of Forecasting*, 27, 870 - 886.

Aruoba. S.B., Diebold F.X., Schorfheide. F., Nalewaik. J. and Song. D. (2011). Improving GDP measurement: A forecast combination perspective. Federal Reserve Bank of Philadelphia working paper.

Bates J.M., and Granger, C.W.J. (1969). The combination of forecasts. *Operational Research Quarterly*, 20(4), 451 – 468.

Bierens, H. (2003). Unit Roots, in Baltagi, B. (editor), *A Companion to Theoretical Econometrics*, Blackwell Publishing.

Bjornland, H.C., K. Gerdrup., A.S. Jore., C. Smith and Thorsrud, L.A. (2010). Does forecast combination improve Norges Bank inflation forecasts? Center for Applied Macroeconomic Research working paper.

Cheng G. and Yuhong, Y. (2015), Forecast combination with outlier protection. *International Journal of Forecasting*, 31, 223 – 237.

Dickey, D.A. (1976). *Estimation and Hypothesis Testing in Non-stationary Time Series*, Ph.D. dissertation, Iowa State University.

Diebold, X. F. (2006). *Elements of Forecasting.* Thomson South-Western Publishing.

Diebold X.F. and Lopez, J.A. (1996). Forecast evaluation and combination. National Bureau of Economic Research working paper series, No. 192.

Diebold, X.F. (1989). Forecast combination and encompassing: Reconciling two divergent literatures. *International Journal of Forecasting,* 5, 589 – 592.

Eklund, J. and Karlsson, S. (2005). Forecast combination and model averaging using predictive measures. Sveriges Riksbank working paper.

Ericsson, N.R. (1989). Parameter constancy, mean squared forecasts errors, and measuring forecast performance: An exposition and extensions. Unpublished manuscript, Federal Reserves Board, Washington, D.C.

Fuller, W.A. (1976). *Introduction to Statistical Times Series.* New York: Wiley.

Hendry, D.F. (2002). Forecast failure, expectations formation and the Lucas Critique. *Annales d'Économie et de Statistique / The Econometrics of policy evaluation.* 67/68, 21 – 40.

Ikoku, A. E. (2014). Modeling and forecasting currency in circulation for liquidity management in Nigeria. *CBN Journal of Applied Statistics,* 5(1), 79 – 104.

Kapetanios, G., J. Mitchell., S. Price and Fawcett, N. (2015). Generalized density forecast combinations. *Journal of Econometrics,* 188(1), 150 – 165.

Newbold, P. and Granger, C.W.J. (1974). Experience with forecasting univariate time series and the combination of forecasts. *Journal of Royal Statistical Society, Series A.* 137(2), 131 – 165.

Panopoulou, E. and Vrontos, S. (2015). Hedge fund return predictability; To combine forecasts or combine information? *Journal of Banking & Finance* (Accepted 2015)

Phillips, P.C. and Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika,* 77, 335 -346.

Samuels, J.D., and Sekkel, R.M. (2013). Forecasting with many models: Model confidence sets and forecast combination. Bank of Canada working paper.