A DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM (DSGE) MODEL OF EXCHANGE RATE PASS-THROUGH TO DOMESTIC PRICES IN NIGERIA

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ABSTRACT

This paper employed a Bayesian framework of DSGE model to estimate the pass-through effect of exchange rate to domestic inflation in Nigeria, using quarterly data for the period 1990 to 2011. The response of inflation rate to exchange rate shocks was found to be positive and statistically significant in the short-run. The pass-through was small and incomplete with almost zero in quarter 1 (0.09), rose to 0.18 in quarter 2 and declined to 0.07 and 0.01 in quarters 3 and 4, respectively. These findings were lower than those obtained elsewhere by other authors. The low pass-through reported in this study was attributed, in part, to the low, stable, and predictable inflation rate arising from the improved credibility of monetary policy environment.

Keywords: DSGE Model, Exchange Rate Pass-through, Inflation, Nigeria

JEL Classification: C01, C51, C87,
INTRODUCTION

Developing economies, like Nigeria, have historically been reluctant to permit more than a moderate degree of exchange rate flexibility due to the pass-through effect of such variations on domestic prices. The potential vulnerability of small and open economies to exchange rate pass-through (ERPT) into domestic prices is high and this arises from the high share of tradable goods, high import content of domestic production and exports, as well as generally high degree of integration with the global trading system.

Policy makers, more often, pay adequate attention to the degree and speed of ERPT into domestic prices. For instance, if pass-through is low, a variation in the exchange rate to improve the trade balance may prove impotent. The implication is that policy makers may not necessarily need to be worried about potential inflationary consequences of exchange rate fluctuations. However, in recent times, there seems to be a growing degree of disconnect between exchange rate changes and domestic consumer prices (Oyinlola, 2011; Oriavwote and Omojimite, 2012; Oladipo, 2012).

The degree and magnitude of ERPT has significant implications for optimal monetary policy transmission mechanism in open economies. It is observed that flexible exchange rate aids relative price adjustment arising from country-specific real shocks. As relative prices changes, an expenditure-switching effect is produced between domestic and foreign goods that are capable of offsetting the initial impact of the shocks. This argument is premised on the fact that domestic prices of imported goods react to changes in nominal exchange rates. Thus, a low degree of ERPT suggests a minimal expenditure-switching, which is capable of shortening the speed of adjustment of nominal exchange rates and hence limiting its suitability as an adjustment mechanism in the short-run.

In the literature, the “new open economy macroeconomics (NOEM)”, with respect to ERPT using the dynamics stochastic general equilibrium (DSGE) models has receive much attention (Bache, 2007; Lane, 2001; Sarno, 2001; and Bowman and Doyle, 2003). Today, DSGE models have become indispensable frameworks for analysing economic policy among academia
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and institutions such as central banks with expectations playing a critical role in decision making effort of monetary authorities (Adebiyi and Mordi, 2010, Bache, 2007). The increasing popularity of these models was attributed partly to their ability to address the Lucas (1976) critique. In response to the Lucas critique, it is argued that policy analysis should be premised on inter-temporal optimising models with explicit micro-foundations (Bache, 2007). DSGE models incorporate economic agents that are forward-looking such that current decisions are not only influenced by expectations of future policies but changes in future expectations also affect current decisions.

Although there are large volumes of work in the literature on DSGE, analysing different areas of economic issues in both developed and emerging economies, only few studies have been conducted on Nigeria (Alege, 2009; Olekah and Oyaromade, 2007; Olayeni, 2009; Garcia, 2009; and Adebiyi and Mordi, 2010). From available information, there are no studies that applied DSGE models to investigate the extent and speed of ERPT to domestic prices. This paper, therefore, aims at filling this gap by adopting Bayesian technique/framework of DSGE to estimate the extent and speed of ERPT to domestic prices in Nigeria.

The structure of the paper is organised as follows. Following the introduction, section 2 provides the theoretical underpinning and literature review. Framework for DSGE modeling is discussed in Section 3, while methodology, covering the data, models set-up and their description are discussed in section 4. Model estimation, using Bayesian technique and the interpretation of the prior and posterior estimates are covered in Section 5. This is followed with the analysis on impulse response functions in Section 6. Section 7 summarises and concludes the paper.

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1 which argued that possibility of parameters stability of coefficients in traditional data-based econometric becomes unlikely due to shift in policy regime
2 For examples Benhabib, Rogerson and Wright (1991) conduct the study for USA; Bergoeing and Soto (2002), for Chile; Kose (1999) and Hofmaiser and Roldos (1997), for Asia; Maussner and Spatz (2005), for Germany; and Christodulakis, Dimeli and Kollintzas (1999), for the European countries.
2.0 THEORETICAL UNDERPINNING AND LITERATURE REVIEW

Under a perfectly competitive market, marginal cost equals price. However, in an imperfectly competitive market, there is possibility of firm earning abnormal profit depending on the degree of market imperfections. In this situation, what determines the variation in markup is the degree of substitutability between domestic and imported goods and this depends on the ability of a firm to differentiate its product and segregate its market. Market segmentation, however, is made possible when geographical location has no systematic effects on transaction prices for identical products (Oladipo, 2006). Product segmentation is geographically possible if the location of the buyers and sellers influences the terms of the transaction substantially.

A market that is integrated may not be perfectly competitive. A monopoly supplier may charge a price above marginal cost, but not able to practice price discrimination if buyers are well organised or if the products are easily transportable across markets. Market power of the sellers is, therefore, greater, the lower the degree of substitutability between domestic and imported goods and lower, the higher degree of market integration.

There is extensive literature on open-economy macroeconomics. However, there are limited studies in the specific area of full-fledged dynamic modeling on exchange rate pass-through. It is more common for studies to follow Monacelli (2005) and introduce Calvo-type importers in model specifications where agents buy goods that are produced domestically and have them sold to foreign countries, even though they face Calvo type pricing frictions and can occasionally optimally reset their prices (Calvo, 1983). A useful example, Smets and Wouters (2002) incorporated the monopolistically competitive importers into a relatively large scale open economy model. Similarly, Lubik and Schorfheide (2006) estimated an open economy New Keynesian model via Bayesian macroeconometric method, incorporating monopolistically competitive firms in a relatively small scale DSGE model.

The results and interpretation in the literature on the estimation of producer currency pricing (PCP) and local currency pricing (LCP) revealed that, there is a lower pass-through for indices defined in terms of import prices for a limited
set of manufactured goods in comparison to homogeneous ones, particularly primary products. In addition, significantly lower estimates of coefficients were reported for consumer price index (CPI) when it was used as the dependent variable, compared with an import price index that is narrower, because of the associated non-tradable nature and base point retail costs. Furthermore, core-CPI measures, rather than the headline measure, decreased the estimates further, since the volatility in prices of raw materials are typically not included, as they are usually the imports with larger pass-through. The estimated pass-through coefficients also differ considerably across countries.

Obstfeld and Rogoff (1995, 1996) are acknowledged as the first to build new open economy macroeconomics models that incorporated preset prices in the currency units of the exporter’s country referred to as PCP. The model presented within the framework of a two-country model, evidence that monetary expansion of a country is always profitable to the partner country. On the other hand, Betts and Devereux (2000) developed a model with the assumption of LCP which is the quotation in importer’s currency and showed that different price setting led to different welfare outcomes.

Parsons and Sato (2006, 2008) examined pass-through effects on Japanese exports in an application with highly disaggregated data. The findings showed that the Japanese exporters seem to fully pass-through exchange rate movements to Vietnamese importers in the machinery industry. However, in the electronic industry, a low degree of exchange rate pass-through appeared prevalent, probably due to US dollar invoicing in trade between Japan and Vietnam. Ito and Sato (2008) studied pass-through in East and Southeast Asia in the post-Asian crisis period using a VAR methodology. The novelty of this paper is in the application of a model-based estimation approach to the issue of pass-through. Campa and Goldberg (2005) studied exchange rate pass-through into import prices for twenty-three OECD countries and the findings revealed that there was evidence in favour of partial pass-through for both PCP and LCP. The paper revealed an average ERPt coefficient of 0.46 in the short-run, and 0.64 in the long-run for import prices, respectively. Shioji, Vu and Takeuchi (2007) developed a Bayesian estimation technique to analyse the partial pass-through using the Japanese
aggregate data. It was found that pass-through was incomplete on both the export and import side of the Japanese economy.

Choudhri and Hakura (2006) estimated the exchange rate pass-through to inflation for 71 countries from the period 1979 to 2000. The average pass-through elasticity for the set of countries classified as low inflation countries was 0.04 per cent in the first quarter, 0.14 per cent after four quarters and 0.16 per cent after twenty quarters. On the average, several countries had negative short-run pass-through elasticities. Comparing regimes across countries and across time, the authors found a significant evidence to show a positive relationship between pass-through and average inflation.

Oyinlola and Egwaikhide (2011) studied exchange rate pass-through to different measures of domestic price in Nigeria by applying a Vector Error Correction model. By employing data from 1980 to 2008, the study revealed that long-run relationship existed between exchange rate and domestic price level. In addition, it was shown that short-run variations in exchange rate might be anticipated and, thus, had its impact dampened.

In their study, Oriawwote and Omojimite (2012) examined the relationship between ERPT and domestic prices in Nigeria using the Vector Error Correction Model. Applying annual data from 1970 to 2009, the study showed that exchange rate volatility induced domestic inflation in Nigeria, and, thus, recommended that exchange rate volatility should be incorporated in the formulation of monetary policy in Nigeria.

Oyinlola (2011) investigated the impact of exchange rate movements on prices of disaggregated imports in Nigeria from 1980 to 2006, taking trade policy into consideration. The outcome of the study revealed that exchange rate exhibits positive and more-than-complete pass-through to import prices of consumer and capital product groups, with mixed interpretations for intermediate products. Hence, depreciation of exchange rate outstrips the impact of tariff reduction on prices of some products.
Oladipo (2012) investigating sectoral exchange rate pass-through effects, revealed that sectoral dependence on imports varied across sectors and showed evidence of incomplete pass-through at varying degrees across sectors. As a result, when adjustment in relative prices is dampened, it reduces considerably the incentive for consumers to switch expenditure from foreign to domestic goods. The implication is that exchange rate policy may not be the most appropriate instrument to be used in dealing with external imbalances.
3.0 Framework of DSGE Modelling

The existing DSGE models in the literature combined the characteristics of both the new-Keynesian (NK) and the real business cycle (RBC) methodologies. The standard DSGE model is micro-founded, open or a closed economy with the reflection of real and nominal rigidities (Christiano, et al., 2005; and Smets and Wouters, 2003). Effort was made to illustrate the basic elements of DSGE models from the view of ‘mathematical language of economists’. These models, though simple, provided a detailed empirical description of the development of output, inflation, and the nominal interest rate in Nigeria. However, some basic features of standard DSGE models were excluded from the model. These included: the process of capital accumulation by firms in the demand block; the detailed treatment of labor market covering employees number of hours of work and the number of people at work; the obstacle to the effective working of financial markets; and the assumption that central banks could fully regulate the short-term interest rate (Sbordone et al., 2010).

3.1 Household

3.1.1 Households and the Aggregate Demand Block

In all DSGE models, consumption decision of households was modeled from the ‘optimal choice of a very large representative household that desired to maximise their expected discounted lifetime utility, looking forward from an arbitrary date t0. Thus, there existed an inverse relationship between the real interest rate and household desired spending as shown in 1(a).

\[
\begin{align*}
    \{X_{t_0:s}, Z_{t_0:s}\}_{t_0:s}^{T_t} & \left[ \prod_{i=0}^{s} \left( \frac{C_{t_0:s} - \eta C_{t_0:s-1}}{C_{t_0:s}} \right) \right] \left( b_{t_0:s} \sum_{t_0:s} \log \left( C_{t_0:s} - \eta C_{t_0:s-1} \right) \right) \ dt 
\end{align*}
\]

subject to the sequence of budget constraints

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3 This sub-section benefited immensely from the work of Sbordone, et al. (2010).
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\[ P_t Z_i + \frac{X_t}{R_t} \leq X_{t-1} + \int_0^t w_i(i) T_i(i) \, di, \]

for \( t = t_0, t_0+1, \ldots, \infty \), and given \( X_{t-1} \).

The representative household prefers more consumption to spending longer hours at work, \( T_t \), as described by the convexity of demand function \( \psi \). The satisfaction derived from consumption is a function of not only the current, but also the past consumption, with a coefficient of \( \eta \). With this ‘habit’, a rational consumer ensures that his current consumption does not fall below his recent past consumption.

In deciding how much to consume, household consumption is obtained by working for a number of hours \( T_i(i) \) in each of the \( i \)-firms, to earn an hourly nominal wage \( W_i(i) \), which is assumed to be given in deciding how much to work\(^4\). The household can use his earned income to buy the final good at price \( P_t \) or save, which can come from accumulating one-period discounted government bonds \( X_t \), with a gross rate of return of \( R_t \) between \( t \) and \( t+1 \).

From time \( t \), utility in time \( t+1 \) is discounted by time-varying factor \( \beta \frac{b_{t+1}}{b_t} \) where \( \frac{b_{t+1}}{b_t} \) is an exogenous stochastic process. A shock to household’s impatience is represented by changes in \( \frac{b_{t+1}}{b_t} \). When \( b_t \) increases faster than \( b_{t+1} \), for instance, the household cares more about the present than the future, and consequently, increases the current consumption relative to the future. Thus, \( \frac{b_{t+1}}{b_t} \) acts as a conventional demand shock, which influences desired consumption and saving exogenously. A persistent decrease in \( \frac{b_{t+1}}{b_t} \) reflects current macroeconomic condition in a country in which households have to reduce their future savings to increase their current consumption. In reality, this observed change in behaviour is explained by many intricate factors, including the concern of people about the future, which is the exclusive focus of this model.

\(^4\) In equilibrium, wage rate is determined at the level at which the supply of labor by the household equals the demand of labor by firms. The demand for labour, in turn, is a function of the need of firms to hire enough workers to satisfy the demand for their products.
The household behaviour in 1[a] is optimised by forming the Lagrangian function:

\[ L = E \sum_{t=0}^{\infty} \left\{ \beta^t \left[ b_{\mu, t} \left( \log(Z_{\mu, t} - \eta Z_{\mu, t-1}) - \int_{v}^{\mu} \left( T_{\mu, v} (i) \right) di \right) - \Lambda_{\mu, t} \left( P_{\mu, t} Z_{\mu, t} + X_{\mu, t} R_{\mu, t} \right) \right] \right\} \]

with first-order conditions

\[ \frac{\partial L}{\partial X_i} : \Lambda_i = \beta E_\mu \left[ \mathcal{L}_i \right] R_i \quad (1\text{ b}) \]

\[ \frac{\partial L}{\partial Z_i} : \Lambda_i P_i = \frac{1}{Z_i - \eta Z_{i-1}} - \eta E_\mu \left[ \frac{\beta b_{\mu, t-1} / b_{\mu, t}}{Z_{i-1} - \eta Z_i} \right] \quad (1\text{ c}) \]

for \( t = t_0, t_1, \ldots, \infty \) and

\[ \frac{\partial L}{\partial T_i (i)} : \Lambda_i / b_i \left[ v^\prime (T_i (i)) \right] = W_i (i) \quad (2) \]

for \( t = t_0, t_1, \ldots, \infty \) and \( \mathcal{X} i \in [0, 1] \) together with the chain of budget constraints, which produced a fully state-contingent plan for the household’s choice variables, and provided an answer to the question on how much to work, consume, and save in the form of bonds. It is to be observed that the household is conscious of the kind of random exogenous outcomes that might influence its actions and the likelihood occurrence of these outcomes.

Consequently, expectations of households about future events are formed, and are rational basis. This implies that expected outcomes are premised on the belief that economic agents are fully informed about the economy and the random exogenous events that affect it.

For example, equation 1 establishes optimality conditions, which shows the negative relationship between the interest rate and desired consumption. This describes the demand side of the model and is clearer in the special case of no habit in consumption \( (\eta = 0) \). The two equations can be combined to obtain the Euler equation.
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\[
\frac{1}{Z_t} = E_t \left[ \frac{\beta b_{t+1}}{b_t} \frac{1}{Z_{t+1}} \frac{R_t}{P_{t+1} / P_t} \right]
\]  

(3)

From this Equation, desired consumption decreases as (gross) real interest rate \( \left( \frac{R_t}{P_{t+1} / P_t} \right) \) increases, anticipated future consumption also falls, and households become more impatient (\( b_{t+1} \) falls).

After some manipulations, a log-linear representation of the Euler equation (3) is expressed as:

\[
q_t = E_t q_{t+1} - (i_t - E_t \pi_t) - \delta_t
\]  

(4)

where \( \pi_t = \log P_t / P_{t+1} \) is the quarterly inflation rate, \( i_t = \log R_t \) represents the continuously compounded nominal interest rate, \( \delta_t = E_t \log (\beta b_{t+1} / b_t) \) is a transformation of the demand shock, and \( q_t = \log Q_t \) stands for the logarithm of total output. Since consumption is the only source of demand for the final good, \( Z \), it is rational to substitute consumption of the final good with its output \( Q \), and thus, market clearing would entail \( Q_t = Z_t \).

In this structure, equation 4 is akin to a conventional investment saving (IS) equation, relating aggregate output, \( y_t \) to the expected real interest rate, \( i_t - E_t \pi_t \). This must exist for the final-good market to clear. However, this equation differs from the conventional IS equation because it is dynamic and forward looking in nature. It shows current and future expected variables. Specifically, it shows a relationship between existing output and the total future expected path of real interest rates as shown in equation 5. This equation reveals the channel through which expectations affect current economic conditions.
\[ q_t = -E_t \sum_{s=0}^{\alpha} (i_{t+s} - \pi_{t+s+1} - \delta_{t+s}) \]  \hspace{1cm} (5)

It should be noted that Equation 4 is a simplified version of the full Euler equation, where the nature of the consumption habit \( \eta \neq 0 \). However, this does not change the qualitative nature of the link between real rates and demand.

Equation 2, which represents the labor supply decision, is the third first-order condition of the household optimisation problem. It shows that workers are willing to work more hours if firms pay a higher wage\(^5\). In reality, a significant increase in wages would generate a positive income effect such that workers who are currently richer as a result of the increase would curtail their labour supply significantly. Technically, workers with higher income tend to raise consumption, thereby causing marginal utility \( \Lambda \), to fall and invariably decrease labor supply at any given wage level significantly.

Thus, labour supply schedule (in equation 2) shows the link between the wages that firms must pay to motivate workers to work a certain number of hours. With rising economic activities, however, firms are willing to pay higher hourly wages that correspond with the desire of the household to work longer hours. All these are critical to the production and pricing decisions of firms, which is the subject of discussion in the next section.

3.1.2 Firms and the Aggregate Supply Block

In the supply block of a DSGE model, firms set their prices with consideration given to the level of demand facing them. Consequently, a positive link exists between price level and real economic activity. From the microeconomic foundations perspective, firms’ structure of production covers a set of monopolistic \( i \)-firms, including an \( f \)-firm that simply sums up the output of the \( i \)-firm into the final consumption good. This section focuses on the firms’

\(^5\) Labor supply is upward sloping because \( v' \) is an increasing function, as \( v \) is convex.
problem with no consideration given to the f-firm. This is due to the fact that all
the pricing system occurs within the i-firms.

It is assumed that intermediate firm \( i \) hires \( T_{ii}(i) \) units of labour of type \( i \) to
produce \( Q_i \) units of intermediate good \( i \) with the technology in a competitive
market.

\[
Q_i(i) = K_i T_{ii}(i)
\]  

(6)

where \( K_i \) stands for the overall efficiency of the production process and is also
assumed to follow “an exogenous stochastic process”, whose random
fluctuations over time reflect the unanticipated changes in productivity.
Assuming a monopolistically competitive market for intermediate goods (Dixit
and Stiglitz, 1977) where firms set prices subject to the condition that they
satisfy the demand for their goods, the demand for f-firm is derived as:

\[
Q_f(i) = Q_f \left( \frac{P_f(i)}{P_i} \right)^{-\theta_t}
\]

(7)

where \( P_{f}(i) \) is the price of good \( i \) and \( \theta_t \) is the elasticity of demand. With
increase in the relative price of good \( i \), its demand falls relative to aggregate
demand by an amount determined by \( \theta_t \).

In the economic literature (Bils and Klenow, 2004; Nakamura and Steinsson,
2008) firms alter their prices occasionally and they do not adjust prices
frequently, but rather hold them constant in some cases for long periods of
time. In line with Calvo (1983) assumption, in every period of time only a
fraction, \( 1 - \alpha \) of firms, is allowed to reset its price while other fraction
maintains old price. The portion that is able to reset their price, set it optimally
at \( t \) and define it as: \( \Omega \subset (0, 1) \), thereby maximising the discounted stream
of expected future profits with the belief that \( s \) periods from now, there is a
likelihood that $\alpha^s$ will be forced to retain their current price. Based on this fact, each of the firms will form its objective function as:

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} \alpha^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_1} \left\{ p_t(i) q_{t+s}(i) - w_{t+s}(i) T_{t+s}(i) \right\}$$

for all $i \in \Omega_t$, subject to the production function in equation 8.

$$q_{t+s}(i) = q_{t+s} \left( \frac{p_t(i)}{p_{t+s}} \right)^{a_{t+s}} \quad (8)$$

for $s = 0, 1, \ldots, \infty$. Profits, defined as total revenue at current price, $p_t(i)q_{t+s}$, less total costs $w_{t+s}(i)T_{t+s}(i)$, are discounted by the multiplier $\beta^s \Lambda_{t+s}/\Lambda_t$, which converts future profits to its current value.

The derivation of the first-order condition (FOC) for optimisation is given as:

$$E \sum_{s=0}^{\infty} (\alpha \beta)^s \Lambda_{t+s} p_{t+s}^{\alpha_{t+s}-1} \left[ p_t(i) - \mu_{t+s} \frac{w_{t+s}(i)}{K_{t+s}} \right] = 0 \quad (9)$$

For all $i \in \Omega_t$, where $p_t'(i)$ is the optimal price chosen by firm i, $w_{t+s}(i)T_{t+s}(i)$ is the nominal marginal cost of the firm at time $t+s$, and $\mu_{t+s} = \frac{\theta_{t+s} - 1}{\theta_{t+s}}$ is its anticipated mark-up given the flexibility of the prices. To maximise profit, a rational monopolist set its price as a mark-up over their marginal cost, but this relationship holds given the expected present discounted value. This is because a price chosen at time $t$ will hold in period $t+s$ with a probability of $\alpha^s$ and thus:
\[ S_{t+s}(i) = \frac{W_{t+s}(i)}{K_{t+s}} = \frac{v'[T_{t+s}(i)]}{K_{t+s}} \frac{1}{N_{t+s}/b_{t+s}} \]

\[ = \frac{v'\left(\frac{Q_{t+s}(i)P_{t+s}(i)}{K_{t+s}}\right)^{\alpha_{t+s}}}{K_{t+s}N_{t+s}/b_{t+s}} \]

From equation 10, wage and production function in equation 6 and the demand function in equation 8 are substituted with the labour supply in Equation 2 to arrive at a term for the labour demand \( T(i) \), which is equivalent for ‘solving’ for equilibrium in the labour market.

The equation for the desired mark-up, \( \mu_{t+s} = \frac{\sigma_{t+s}}{\sigma_{t+s}} - 1 \) says that a monopolist that faces an inelastic demand charges a higher mark-up, which translates to higher price since the consumers are indifferent to price increase. This insensitivity is assumed to follow an exogenous stochastic process. A positive shock to desired mark-up, for instance, increases the firm’s market power, which invariably translates into higher prices.

From Equation 9, if aggregate price level is set as a function of newly set prices \( P^* \) and the past price index \( P_{t-1} \), then:

\[ P_t = \left[ (1 - \alpha) P_t^{(1-\alpha)} + \alpha P_{t-1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \]

produces an estimated New Keynesian Phillips (NKP) curve, connecting current inflation with future expected inflation and real marginal cost as follows:

\[ \pi_t = \xi \xi_t + \beta E \pi_{t+1} + u_t \]

where \( u_t = \xi \log u_t \) is a transformed mark-up shock and \( j = \log(S/P) \) is the logarithm of the real marginal cost (RMC). The responsiveness of inflation to changes in the MC, \( \xi \), is a function of the rate of price change, \( \alpha \) and
other structural parameters as indicated by 

\[ \xi = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha (1 + \omega \theta)} \], where

\[ \omega = \frac{v^T}{v} \] is the elasticity of the marginal disutility of work, while \( \theta \) is the elasticity of demand’s average value.

Supply block, is defined by the Phillips curve in equation 11, which shows that marginal cost is a function of the level of aggregate activity, among other factors. Higher economic activity translates to higher wages, higher marginal cost and higher inflation.

Another characteristic of the Phillips curve is that it is forward looking. By iterating equation 11 forward, we obtain:

\[ \pi_t = E_t \sum_{s=0}^{\infty} \beta^s \left( \xi s_t + u_{t+s} \right), \]

which reveals how current inflation is explained by the expectation of MCs and output, and invariably depends on the expected interest rates and expected monetary policy action as shown in equation 5.

3.1.3 Monetary Policy

In Equation 5, it is shown that low interest rate —current and expected, encourages more consumption of goods. However, with high demand, firms’ marginal costs and prices tend to increase, thereby raising inflation. The converse holds when the interest rate is high. It should be noted, however, that short-term interest rate enters the models through the action of the monetary authority that sets the nominal interest rate. In Nigeria, this is a decision made by the Monetary Policy Committee (MPC) using various input from the Monetary Policy Technical Committee (MPTC), simulation of models using different scenarios for forecasting, and the opinion of the policymakers, among others. Notwithstanding the noticeable difficulty of the process, Taylor (1993) has clearly shown that it could be logically demonstrated with the assumption that central banks, and in our case, the Central Bank of Nigeria
(CBN), raises the monetary policy rate when inflation and/or output is “high” with reference to the baseline. This is an expected behaviour in almost all segments of DSGE models, but with contentious issue regarding the correct baselines.

In this model, therefore, it is assumed that interest rates are set based on policy rule

$$i_t = \rho r^*_t + \left(1 - \rho \right)
\left[r^*_t + \pi^*_t + \phi (\pi^{py}_t - \pi^*_t) + \varphi_i (q^*_t - q^*_t)\right] + \epsilon_i^t
$$

(12)

Where $r^*_t$, $\pi^*_t$, and $q^*_t$ are the baselines for the real interest rate, inflation, and output, respectively, and $\pi^{py}_t = \log \left(\frac{p_t}{p_{t-4}}\right)$ is the inflation rate for the preceding four quarters. The monetary policy shock $\epsilon_i^t$ captures any discrepancy between the perceived nominal interest rate and what the rule suggested. This rule implies that if inflation and output rise above their baseline levels, the nominal interest rate is raised over and above the baseline, $r^*_t + \pi^*_t$, by values determined by $\phi$, and $\varphi_i$ and at a speed of $\rho$. The higher policy rate, which is projected to persist after output and inflation have returned to the steady state, restrains the economy, thereby reducing demand, marginal costs, and inflation. Thus, $\pi^*_t$, and $q^*_t$ is seen as targets of monetary policy that is consistent with the mandate of the CBN.

Production of equilibrium or ‘efficient’ level of output could be identified as part of the central bank’s objective and could be represented as $z_t$. This unobserved variable represents the prevailing output level that is capable of eliminating all distortions in the economy. The level of activity resulting from such behaviour is expected to be realistic from the view of the representative household in the model and makes it an appropriate target for monetary policy. When output is efficient, inflation is unstable and fluctuates due to the presence of mark-up shocks. Thus, explaining the monetary policy trade-offs at any particular point in time.
4.0 METHODOLOGY

4.1 Model Set Up and Description

Generally, DSGE models have the characteristics of the NK paradigm and the RBC approach, fully micro-founded with real and nominal rigidities (see for instance Christiano, et al., 2005; and Smets and Wouters, 2003). Considering the peculiarities of Nigeria as an oil-dependent economy, the dynamic evolution of the endogenous variables of interest in the Nigerian economy are as explained in equations 13 to 16: aggregate demand equation (IS curve), aggregate supply (the Phillips curve), uncovered interest rate parity (UIP) and a forward-looking Taylor rule.

The dynamic evolution of the endogenous variables is explained in four equations covering a period of 1990:1 – 2011:4. The variables are expressed in differenced forms as specified in equations 13 to 16:

\begin{align}
\Delta y_g_t &= a_1 y_{g,t-1} + a_2 y_{g,t-1} - a_3 m_{ci,t} + a_4 g_{ov,t} + \varepsilon_{yg} \\
\Delta m_{ci,t} &= a_5 z_{t} + (1 - a_4) \varepsilon_{z} \\
\Delta \inf_t &= b_1 \inf_{t-1} + (1 - b_2) \inf_{t-1} + b_3 \rmc + b_4 p_e + b_5 m_2 + \varepsilon_{\inf} \\
\Delta s_t &= e_1 s_{t-1} + e_2 y_{g,t} - e_3 \res - e_4 (i_{t-1} - i_t) + \varepsilon_{z} \\
\Delta i_t &= f_1 i_{t-1} + (1 - f_1) (i_{t-1}^* + f_2 (\inf_{t-1} - \inf^*) + f_3 y_{g,t} + f_4 \res + \varepsilon_i) \\
\Delta pf_t &= t_1 pf_{t-1} + \varepsilon_{pf} \\
\Delta i_t^* &= t_1 i_{t-1}^* + \varepsilon_{i_t} \\
\Delta ygf_t &= t_1 ygf_{t-1} + \varepsilon_{ygf} \\
\Delta pof_t &= t_1 pof_{t-1} + \varepsilon_{pof} \\
\Delta gov_t &= t_1 gov_{t-1} + \varepsilon_{gov} \\
\Delta res_t &= t_1 res_{t-1} + \varepsilon_{res} \\
\Delta m_{tr} &= t_1 m_{tr,t-1} + \varepsilon_{m_{tr}}
\end{align}

Benefited immensely from the work of Adebiyi and Mordi (2010)
where: \( y_g_t \) is the output gap in time \( t \); \( y_g_f \) is the foreign output gap in time \( t \); \( mci \) explains the real marginal condition index in time \( t \); \( z_i \) is the real exchange rate in time \( t \) defined as nominal exchange rate deflated by relative prices; \( \Delta z_i \) is the change in the equilibrium exchange rate in time \( t \); \( s_i \) is the nominal interest rate in time \( t \); \( gov \) stands for the government total expenditure; \( inf_i \) represents inflation rate in period \( t \); \( rmc \) is real marginal cost in time \( t \); \( inf_{t+1}^e \) is the expected inflation rate in time \( t \); \( inf_t^e \) stands for optimum or equilibrium inflation rate in time \( t \); \( prem_i \) stands for exchange rate premium in time \( t \); \( i \) is the domestic nominal short-term interest rate in time \( t \); \( po \) is the oil price (bonny light); \( m_2 \) stands for the broad money supply; \( i_f \) is the foreign nominal short-term interest rate in time \( t \); \( i_n \) represent the natural rate of interest in time \( t \); \( t-i \) represents the lags of relevant variables; \( t+i \) stands for the lead relevant variables; and \( a, b, e \) and \( f \) are all parameters to be estimated.

Equation 13 is an enriched version of the standard new-Keynesian Euler equation for consumption, which is theoretically linked to household utility optimisation. According to the theory, household maximises discounted stream of utility (consumption and labor supply) subjected to budget constraints (consumption expenditure and wages). In calculating the present value of spending and wages, interest/policy rate is incorporated in the equation.

The lag of output gap (\( y_g_{t-1} \)) is included in equation 13 to give room for some degree of habit persistence in consumption or adjustment costs of investment (Pongsaparn, 2008).

Nigeria is a small open economy and consequently, real exchange rate gap (\( z \)) was included as a variable that influences economic activities through the prices of imports and exports. The relative weight of the real interest and real exchange rates is explained by a monetary condition index (MCI) in the IS curve. Also, foreign output gap (\( y_g_f \)) was added as a determinant of export demand. The influence of other explanatory variables such as oil price, fiscal policy and other demand shocks are captured by the residual term.

Equation 14a is the inflation equation specified in the spirit of the Philips curve. The equation shows that inflation rate is influenced not only by past inflation,
but also by inflation expectations, demand pressures, and external supply shocks captured by \( z_t \). From this equation, inflation depends on its expected future value and its own lagged value. The inclusion of the lagged term shows the existence of a short-run trade-off between output and inflation. In the specification of inflation equation, exchange rate effect on domestic prices is considered. The inclusion of the real exchange rate attempts to capture the exchange rate pass-through to domestic prices due to the openness of the economy. Domestic sources of inflation are captured by the inclusion of output gap, \( y_g \). The relative weight of output gap and real exchange rate gap in the firm’s real marginal costs is denoted by \( b_2 \).

Equation 15 is the uncovered interest parity (UIP) equation for an open economy, like Nigeria. \( i_t \) and \( i_f^t \) are the domestic nominal and foreign short-term interest rates, respectively. In the literature, many models that assume interest parity condition do not provide enough persistence to generate a hump-shaped response of the real exchange rate after a shock to monetary policy, which is commonly found in estimated VARs (Eichenbaum and Evans, 1995; Faust and Rogers, 2003). Given the degree of openness of the Nigerian economy, it is plausible to assume that interest parity condition holds in Nigeria. Thus, nominal exchange rate depends on its lead value.

Equation 16 is the modified Taylor’s rule, which explains the interest rate path for the monetary authority. From the equation, monetary authorities react immediately to the changes in inflation and output gaps by altering its monetary policy rate to stabilise both the nominal and real exchange rates. The exchange rate plays an important role in aggregate demand through its effects on net export and also on inflation through the pass-through effect. The UIP shows the link between exchange and interest rates. In reaction to a depreciation of the exchange rate, for example, the monetary authority is expected to raise interest rates, thereafter.
5.0 EMPIRICAL METHODOLOGY

5.1 Bayesian Estimation
This paper employed a Bayesian methodology to estimate equations 13 - 23. Technically speaking, Bayesian estimation is a mix between calibration and maximum likelihood, which are connected by Bayes’ rule. The calibration part is the specification of priors and the maximum likelihood approach enters through standard econometrics based on adjusting the model with data. This methodology has recently been used extensively in estimating complex stochastic models involving very large number of parameters. In such cases, it is typical to conduct the Bayesian estimation via Markov-chain Monte-Carlo (MCMC) simulation rather than the straightforward maximum likelihood estimation. This is because in most of such cases it is not possible to specify the joint distribution of parameters in an explicit manner. This paper adopts the Metropolis-Hastings (MH) algorithm, which is one of the oldest among the existing MCMC sampling methods.

The basic idea of the Bayesian estimation can be summarised as follows:

\[ p(\lambda_{tr}, K_T, R) = \frac{p(K_T | \lambda_{tr}, B) p(\lambda_{tr} | R)}{p(K_T | R)} \]

where \( p(K_T | R) \) is the marginal density of the data condition on the model, \( p(\lambda_{tr} | R) \) represents the priors density function and \( p(K_T | \lambda_{tr}, R) \) is the likelihood function.

5.2 Prior Distributions of the Estimated Parameters
The starting point of the Bayesian inference is the identification of prior distribution, which describes the available information prior to observing the data used in the estimation. In the calibration of the model, the validity of economic theories, stylized facts about the Nigerian economy and observations facts and existing empirical literature are taken into consideration. Thus, the coefficients in Table 1 were obtained. The Phillips curve and the IS curve estimates were obtained from the work of Adebiyi and
Mordi (2010). The estimates of log of output gap of 0.72 was consistent with what was found in Laxton and Scott (2000), who claimed that the sum of the parameters of real interest rate and real exchange rate should be smaller than that of the output gap, owing largely to the limited effect of the interest rate and exchange rate on output due to significant lags in monetary transmission mechanism in most economies. All exogenous variables were assumed to follow AR (1) process, as in (Adebiyi and Mordi, 2010).

Table 1: The Model Calibration and Parameterisation

<table>
<thead>
<tr>
<th>EQUATION</th>
<th>PARAMETER</th>
<th>DEFINITION</th>
<th>VALUE</th>
<th>COMMENTS</th>
<th>LINEAR HOMOGENEITY CONDITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS Curve (Output Gap)</td>
<td>( \alpha_1 )</td>
<td>Lag of output gap</td>
<td>0.72**</td>
<td>Measures output gap persistence; lies between 0.1 and 0.95</td>
<td>0 &lt; ( \alpha_1 ) &lt; 1</td>
</tr>
<tr>
<td></td>
<td>( \alpha_2 )</td>
<td>Marginal condition index</td>
<td>-0.10*</td>
<td>Measures the pass-through from monetary condition to the real economy. It varies between -0.1 to -0.5</td>
<td>(-0.1 &lt; \alpha_2 &lt; -0.5)</td>
</tr>
<tr>
<td></td>
<td>( \alpha_3 )</td>
<td>Foreign output gap</td>
<td>0.12*</td>
<td>Measures the impact of foreign demand on domestic output; varies between 0.1 (low impact) to 0.7 (strong impact)</td>
<td>0 &lt; ( \alpha_3 ) &lt; 1</td>
</tr>
<tr>
<td></td>
<td>( \alpha_4 )</td>
<td>Real exchange rate gap</td>
<td>0.5*</td>
<td>Shows the relative weight of the real interest and exchange rates in real monetary condition;</td>
<td>0 &lt; ( \alpha_4 ) &lt; 1</td>
</tr>
</tbody>
</table>

\( (1a) \)

\( (1b) \)
## Dynamic Stochastic General Equilibrium (DSGE) Model of Exchange Rate Pass-through To Domestic Prices In Nigeria

### Phillips Curve

\[ \inf_t = b_1 \inf_{t-1} + (1 - b_1) \inf_{t-1} + b_2 \text{mc}_t + b_3 \text{mc}_{t-2} + b_4 \text{mc}_t + e_{inf} \quad (2a) \]

\[ \text{mc}_t = b_5 \text{gg}_t + (1 - b_5) z_t \quad (2b) \]

- \( b_1 \): Lag of inflation rate, \( 0.62^{**} \)
  - Measures inflation persistence. It varies between 0.4 (low persistence) to 0.9 (high persistence)

- \( b_2 \): Real marginal costs, \( 0.31^{**} \)
  - Pass-through from real marginal cost to inflation. It measures sacrifice ratio. It varies from 0.05 to 0.4

- \( b_3 \): Exchange rate changes, \( 0.70^{**} \)
  - Ratio of domestically produced goods in the consumer basket. It varies between 0.9 and 0.5

### Uncovered Interest parity

\[ s_t = a_1 s_{t-1} + a_2 y_{g_t} - e_{y_t} e_s - a_4 (z_t - z_{t-1}) + e_s \quad (5) \]

- \( a_1 \): Lag of expected exchange rate, \( 0.1^{**} \)
  - Captures either exchange rate persistency or central bank's interventions: varies between zero to 0.9 (tight control of the exchange rate)

- \( a_2 \): Lag of monetary policy rate, \( 0.70^{**} \)
  - Policy persistence, value varies from 0 (no persistence) to 0.8 ("wait and see" policy)

- \( a_3 \): Deviation of inflation from potential, \( 1.50^{**} \)
  - Measures the weight put on inflation by the policy

---

**Table:**

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phillips Curve</td>
<td>( b_1 )</td>
<td>Lag of inflation rate</td>
<td>0.62^{**}</td>
<td>Measures inflation persistence. It varies between 0.4 (low persistence) to 0.9 (high persistence)</td>
</tr>
<tr>
<td></td>
<td>( b_2 )</td>
<td>Real marginal costs</td>
<td>0.31^{**}</td>
<td>Pass-through from real marginal cost to inflation. It measures sacrifice ratio. It varies from 0.05 to 0.4</td>
</tr>
<tr>
<td></td>
<td>( b_3 )</td>
<td>Exchange rate changes</td>
<td>0.70^{**}</td>
<td>Ratio of domestically produced goods in the consumer basket. It varies between 0.9 and 0.5</td>
</tr>
<tr>
<td>Uncovered Interest parity</td>
<td>( a_1 )</td>
<td>Lag of expected exchange rate</td>
<td>0.1^{**}</td>
<td>Captures either exchange rate persistency or central bank's interventions: varies between zero to 0.9 (tight control of the exchange rate)</td>
</tr>
<tr>
<td></td>
<td>( a_2 )</td>
<td>Lag of monetary policy rate</td>
<td>0.70^{**}</td>
<td>Policy persistence, value varies from 0 (no persistence) to 0.8 (&quot;wait and see&quot; policy)</td>
</tr>
<tr>
<td></td>
<td>( a_3 )</td>
<td>Deviation of inflation from potential</td>
<td>1.50^{**}</td>
<td>Measures the weight put on inflation by the policy</td>
</tr>
</tbody>
</table>

---

**Notes:**

- The lag values are significant at the 1% level.
- The parameters are constrained to be between 0 and 1 (\( 0 < b_i < 1 \), \( 0 < a_i < 1 \)).
A Dynamic Stochastic General Equilibrium (DSGE) Model of Exchange Rate Pass-through To Domestic Prices In Nigeria

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_3$</td>
<td>Output gap</td>
<td>0.50**</td>
<td>measures the weight put on the output gap by the policy maker; value has no upper limit but must always be higher than 0 (the Taylor principle)</td>
</tr>
<tr>
<td>$f_4$</td>
<td>Changes in Exchange rate</td>
<td>0.25**</td>
<td>measures the weight put on the exchange rate by the policy maker; value has no upper limit, but must be always higher than 0</td>
</tr>
</tbody>
</table>

**Note**: * the values are obtained from expert judgment (see JVI/IMF Institute, (2010)).
** the values are obtained from Adebiyi and Mordi (2010)

5.3. **Posterior Distributions of the Estimated Parameters**
To sample from the posterior, random walk Metropolis-Hastings (MH) algorithm was utilised to produce 100,000 draws from the posteriors. The Estimation results are reported in Table 2 and Figure A.1 (in the Appendix). The results showed the distribution used, the prior mean, the prior standard deviation, and the confidence interval.
Table 2: Prior and Posterior Distribution of the Estimated Parameters

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>DESCRIPTION</th>
<th>DENSITY</th>
<th>PRIOR MEAN</th>
<th>POSTERIOR MEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>Measures output gap expectation</td>
<td>Beta</td>
<td>0.35</td>
<td>0.29</td>
</tr>
<tr>
<td>$a_{11}$</td>
<td>Measures output gap persistence</td>
<td>Beta</td>
<td>0.65</td>
<td>0.74</td>
</tr>
<tr>
<td>$a_2 * a_4$</td>
<td>Measures impact of exchange rate on output</td>
<td>gamma</td>
<td>0.40</td>
<td>0.20</td>
</tr>
<tr>
<td>$a_2 * (1-a_4)$</td>
<td>Measures impact of interest rate on output</td>
<td>gamma</td>
<td>0.25</td>
<td>0.13</td>
</tr>
<tr>
<td>$a_3$</td>
<td>Measures the impact of foreign demand on domestic output</td>
<td>Beta</td>
<td>0.50</td>
<td>0.68</td>
</tr>
<tr>
<td>$a_5$</td>
<td>Measures impact of government expenditure on output</td>
<td>gamma</td>
<td>0.20</td>
<td>0.27</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Inflation expectation</td>
<td>Beta</td>
<td>0.30</td>
<td>0.40</td>
</tr>
<tr>
<td>$b_{33}$</td>
<td>Measures inflation persistence</td>
<td>Beta</td>
<td>0.65</td>
<td>0.74</td>
</tr>
<tr>
<td>$b_2 * b_3$</td>
<td>Measures sacrifice ratio</td>
<td>Beta</td>
<td>0.30</td>
<td>0.21</td>
</tr>
<tr>
<td>$b_2 * (1-b_3)$</td>
<td>Measures exchange rate pass-through</td>
<td>Beta</td>
<td>0.25</td>
<td>0.096</td>
</tr>
<tr>
<td>$b_4$</td>
<td>Measures impact of oil price (bonny light) on inflation</td>
<td>gamma</td>
<td>0.20</td>
<td>0.28</td>
</tr>
<tr>
<td>$b_5$</td>
<td>Measures the impact of money supply on output</td>
<td>gamma</td>
<td>0.50</td>
<td>0.37</td>
</tr>
<tr>
<td>$f_1$</td>
<td>Measures policy persistence</td>
<td>Beta</td>
<td>0.20</td>
<td>0.15</td>
</tr>
<tr>
<td>$f_2$</td>
<td>Measures the weight put on inflation by policy makers</td>
<td>Beta</td>
<td>1.50</td>
<td>1.45</td>
</tr>
<tr>
<td>$f_3$</td>
<td>Measures the weight put on output gap by policy makers</td>
<td>gamma</td>
<td>0.50</td>
<td>0.51</td>
</tr>
<tr>
<td>$f_4$</td>
<td>Measures the weight put on exchange rate by policy makers</td>
<td>gamma</td>
<td>0.25</td>
<td>0.23</td>
</tr>
<tr>
<td>$e_1$</td>
<td>Measures exchange rate expectation</td>
<td>gamma</td>
<td>0.50</td>
<td>0.52</td>
</tr>
<tr>
<td>$e_2$</td>
<td>Measures the impact of output gap on nominal exchange rate</td>
<td>gamma</td>
<td>0.20</td>
<td>0.12</td>
</tr>
<tr>
<td>$e_3$</td>
<td>Measures the impact of external reserves on nominal exchange rate</td>
<td>gamma</td>
<td>0.20</td>
<td>0.14</td>
</tr>
<tr>
<td>$e_4$</td>
<td>Measures the impact of interest rate differential on nominal exchange rate</td>
<td>gamma</td>
<td>0.50</td>
<td>0.48</td>
</tr>
</tbody>
</table>
A Dynamic Stochastic General Equilibrium (DSGE) Model of Exchange Rate Pass-through To Domestic Prices In Nigeria

| τ_6  | Measures the AR(1) of foreign (USA) price | gamma | 0.40 | 0.32 |
| τ_7  | Measures the AR(1) of foreign interest rate | gamma | 0.50 | 0.47 |
| τ_8  | Measures the AR(1) of oil price | Beta | 0.45 | 0.42 |
| τ_9  | Measures the AR(1) of money supply | Beta | 0.55 | 0.51 |
| τ_10 | Measures the AR(1) of US GDP | Beta | 0.60 | 0.50 |
| τ_11 | Measures the AR(1) of external reserves | gamma | 0.60 | 0.55 |
| τ_12 | Measures the AR(1) of monetary policy rate | gamma | 0.50 | 0.65 |

* Metropolis-Hastings sampling algorithm based on 100000 draws with 59% acceptance rate.

From Table 2, it is observed that monetary policy influences inflation through its effects on output and the exchange rate. The posterior estimate of the output gap, which measures the sacrifice ratio \( b_2 * b_3 = 0.21 \) is not too far from the prior (0.30). This makes it possible for monetary authorities (particularly the Central Bank of Nigeria) to control inflation through output gap. Also, the impact of the exchange rate on prices \( b_2 * (1 - b_3) = 0.10 \), indicating a low and incomplete pass-through into prices (that is 10 per cent per quarter or 40 per cent per annum).

The estimated exchange rate pass-through in Nigeria (9.6 per cent) when compared with past studies on the subject indicated that exchange rate pass-through to inflation is declining in Nigeria. For example, Aliyu, et. al., (2007) obtained an ERPT of 10.5 per cent; Barhoumi (2007), 14.7 per cent; Garcia (2010), 10 per cent; Oyinlola (2011), 18-47 per cent; CBN (2011), 0.25 per cent; and Oriavwote and Omojimite (2012), 16 per cent. However, these findings should be interpreted with caution since past studies on this subject vary from scope to methodology. Some used annual data, while others employed quarterly data. Apart, while some studies employed ordinary least square (OLS) technique, others used VAR methodologies.
Notwithstanding, some reasons were attributed to the decline in exchange rate pass-through in Nigeria, which included changes in Nigeria’s monetary policy in the late 1980s and early 2000s. This has been responsible for the decline in the exchange rate pass-through into the Nigerian consumer prices during the last 14 years. The introduction of monetary policy rate (MPR) as an anchor rate seems to have had a significant effect on the way the consumer price index responds to technology shocks, relative to the way it responded in the pre-MPR period. Second in the post-MPR era, inflation rate in Nigeria has remained low, stable, and predictable. Inflation expectations have been well-anchored, policy credibility has been enhanced, and the persistence of inflation has been significantly reduced. Third, the propagation of exchange rate shocks is minimised and consequently, exchange rate pass-through is less significant because of the increased confidence on the part of economic agents. Confidence is built on the fact that monetary authority will not allow inflation to move persistently above optimum and will anchor inflation expectations from becoming extrapolative. Lastly, ERPT seems to be declining in Nigeria due to the change in structure and the composition of trade in Nigeria.

Other empirical findings that are relevant to the study are as follows. Change in oil price has a significant impact on inflation with a posterior value of 0.28 (that is $b_4=0.28$), which is very close to the prior values of 0.20. This implies that a 1.0 per cent increase in oil price would raise prices by only 0.28 basis points the following period. The output cost of disinflation, which is the sacrifice ratio, is estimated to be 1.217.

In the hybrid Phillip equation, the hypothesis that the values of forward-looking inflation expectation must be significantly below 0.50 to produce results that is consistent with data is established as in Berg, Karam and Laxton, 2006. The posterior estimates showed that the data provided useful information in explaining inflation behaviour in Nigeria. The behaviour of the economy depends critically on the value of $b_1$. The posterior estimates of Calvo price

---

7 Sacrifice ratio is defined as the cumulative output losses associated with a permanent one percentage point decline in inflation.
stickiness provide reasonable notion about frequencies of price change, which is the probability of not changing price in a given quarter. The estimated value \( b_1 = 0.40 \) shows that domestic firms re-optimise their prices in almost every one and half quarters\(^8\), which is very close 0.382 obtained by Adebiyi and Mordi (2010) and the 0.57 obtained by Garcia (2009) and 0.38 by CBN (2011).

From Table 2, the posterior value \( f_1 = 0.15 \) explained the possibility that the Central Bank of Nigeria can moderate interest rates and adjust them fairly slowly to the desired value based on the deviation of the inflation and output from equilibrium. The weight attached to inflation showed a posterior value of 1.45, which is very close to a prior of 1.50 and this supported the literature that a stable inflation rate requires a positive \( f_2 \). (Berg, Karam and Laxton, 2006). The posterior estimates for output gap and exchange rate were almost pinned down by the data. The posterior mean of exchange rate \( f_4 \) is 0.23, which indicated that monetary authority in Nigeria takes cognisance of exchange rate behaviour when determining the monetary policy rate. For example, if nominal exchange rate is appreciated by 1.0 per cent, maximum interest rate will decline by 23 basis points. This implies that the Taylor-type policy rule for Nigeria is a monetary policy rule that attaches weights not only to inflation and output gap, but also to exchange rate.

With regards to the persistence parameters of the AR(1) process, all of the parameters, except MPR, showed a posterior mean smaller than the mean of the prior. This indicated that the persistence of the shocks is smaller than our prior beliefs and is consistent with the finding reported by CBN, 2011.

\[ f_1 = \frac{1}{1 - b_1} \]

\(^8\) This is obtained as \( \frac{1}{1 - b_1} \)
6.0 PROPAGATION OF SHOCKS

From Figure 1, the response of inflation rate to exchange rate shocks is positive and statistically significant in the short-run. The pass-through was almost zero in quarter 1 (0.09), rose to 0.18 in quarter 2 and declined to 0.07 and 0.01 in quarters 3 and 4, respectively. The finding showed a small and incomplete pass-through of exchange rate to inflation in Nigeria.

In the Figure also, a positive shock to the exchange rate led to a depreciation of the naira. Depreciation encourages exports and discourages imports, thereby causing an immediate increase in output gap, interest rate and inflation. The speed of reversion to steady state, arising from the shock, was about 2-3 years (8-12 quarters) for most of the variables.

Figure 1: Response of Output and Prices to 1% Exchange Rate Shock

Source: Authors’ Calculations

A supply shock as shown in Figure 2 caused an increase in inflation that caused the output gap to decline (i.e. produced a recession). This arose from the need of the monetary authority (Central Bank of Nigeria) to raise the
interest rate to reduce inflation to its original level. As inflation declined, exchange rate depreciated due to the decrease in interest rate. Consequently, in the long-run, all the variables were restored to their steady state values.

Figure 2: Response of Prices and Output to 1% Aggregate Supply Shock

Source: Authors’ calculations

Positive shock to oil price produced an appreciation of the real exchange rate, which arose from the response of the monetary authority to raise interest rate (see Figure 3 and equation 16). Consequently, the appreciation caused a reduction in inflation rate (equation 14a). The impact of the oil price shock on the inflation rate can be viewed from two perspectives. One, the appreciation reduced inflation rate. However, a second-round effect (though not shown here) showed that when the price of oil (bonny light) increased, foreign inflation tended to rise, which caused the domestic inflation rate to increase. Thus, the net effect was that inflation initially fell (because the positive effect of appreciation on inflation exceeded the negative impact of
imported inflation), before it increased, reaching its maximum in quarter 5 and, thereafter, decelerated and returned to its steady state.

**Figure 3: Response of Prices and Output to 1% Oil Price Shock**

![Graphs showing Inflation, Output, Interest Rate, and Nominal Exchange Rate](image)

**Source:** Authors’ calculations

The aggregate demand shock, which is in line with the prediction of the economic theory, produced an expansion of the economy (output) and an increase in the inflation rate in the medium-term (Figures 4). Increase in inflation rate raised the interest rate, which resulted in the appreciation of exchange rate and reduction in output gap in the medium to long-run. However, in the long run, all the variables were restored to their equilibrium state values.
According to economic theory, an increase in monetary policy rate (shock to interest rate) (Figure 5) is expected to decrease output gap and the inflation rate, which is consistent with the findings. This arises due to the fact that the real interest rate is negatively correlated with output gap (see equation 13a), which results to a decline in the inflation rate as shown in the Phillip curve equation 14a. The existence of partial uncovered interest rate parity (that is, the higher interest rate generates capital inflow that caused the naira to appreciate as shown in equation 15) made the shock to cause an appreciation of the naira. Thereafter, all variables returned to their steady state values as the decline in output and inflation generated a downward adjustment in the interest rate as shown in equation 16.
Figure 5: Response of Price and Output to 1% Positive Interest Rate Shock

Source: Authors’ calculations
7.0 CONCLUSION
This study employed a Bayesian framework of DSGE model to estimate the pass-through effect of exchange rate to domestic prices in Nigeria, using a quarterly data for the period 1990 to 2011. The response of inflation rate to exchange rate shock was found to be positive and statistically significant in the short-run. It showed a small and incomplete pass-through of exchange rate to domestic inflation with pass-through almost equal to zero in quarter 1 (0.09), rose to 0.18 in quarter 2 and declined to 0.07 and 0.01 in quarters 3 and 4, respectively. This was lower than the findings reported by previous studies in the area by other authors, Aliyu, et al., (2007) obtained an ERPT of 0.11 per cent; Barhoumi (2007), 0.15 per cent; Garcia (2010), 0.10 per cent; Oyinlola (2011), 0.18-0.47 per cent; CBN (2011), 0.25 per cent; and Oriavwote and Omojimite (2012), 0.16 per cent. An improved monetary policy and enhanced credibility have played an important role in the decline, along with significant shifts in the composition of trade and increased globalisation.
A Dynamic Stochastic General Equilibrium (DSGE) Model of Exchange Rate Pass-through To Domestic Prices In Nigeria
REFERENCES


Cycles in the European Community: Idiosyncracies and Regularities. Economica, 62 (February) 617-52


Laxton, D. and A. Scott (2000). On developing a structured forecasting and policy analysis system designed to support inflation-forecast targeting (IFT), in Inflation Targeting Experiences: England, Finland, Poland, Mexico, Brazil, Chile (Ankara: The Central Bank of the Republic of Turkey), 6-63.


Oyinlola, M.A (2011) Exchange Rate and Disaggregated Import Prices in Nigeria. Journal of Economic and Monetary Integration, Vol. 9, Number 1, pp 89-126


Appendix

Figure A.1: Prior and Posterior Distributions

[Graphs showing prior and posterior distributions for SE_e1 to SE_e9, beta1 to beta4]
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