ARFIMA Modelling and Investigation of Structural Break(s) in West Texas Intermediate and Brent Series

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The research used a long memory or Autoregressive Fractionally Integrated Moving Average model to study and forecast crude oil prices using weekly West Texas Intermediate and Brent series for the period 15/5/1987 to 20/12/2013. Fractional differencing Methods such as Local Whittle Estimator and Geweke and Porter-Hudak identified long memory characteristics in the crude oil prices. For WTI series, the Bayes Information Criteria selected 3 breaks with the first, second and last breaks captured in 1999, 2004 and 2008 respectively. Three breaks in Brent series using the Bayes Information Criteria were selected and this pointed out that Brent series has break points in 1999, 2005 and 2009. Numerous ARFIMA models were identified, selected using Akaike Information Criterion, estimated/check, in sample and out sample forecast was carried out using Box and Jenkins methodology. ARFIMA(1,0.47,2) is appropriate for West Texas Intermediate series while ARFIMA(2,0.09,0) is suitable for Brent series. One year in sample forecast indicates a small difference between the original series and the forecast results. The one year out sample forecast revealed a decline in future crude oil prices which may be good news to the consumers and bad news to the producers.

Keywords: ARFIMA model, Structural breaks, Long memory, Local Whittle Estimator and Crude oil prices.

JEL Classification: C22, C50, D12

1.0 Introduction

The autocorrelation function (ACF) for a stationary time series decays exponentially to zero as lag increases while non-stationary series may have sample autocorrelation function converges to unity for all lags as the sample size increases (Chan and Wei, 1988; Tiao and Tsay, 1983). In most of the micro economic series, ACF decays slowly to zero at a polynomial rate as the lag increases. These types of processes are often called long range dependent or long memory time series. The examination of ACF of a series can serve as

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a prelude to study long memory characteristics. Long-memory processes are stationary processes whose autocorrelation functions decay more slowly than short-memory processes. Nesrin (2006) explained that an autoregressive fractionally integrated moving average (ARFIMA) model provides a parsimonious parameterization of long-memory processes. The ARFIMA model allows for a continuum of fractional differences, $-0.5 < d < 0.5$ while the generalization to fractional differences allows the ARFIMA model to handle processes that are neither I(0) nor I(1) and that ARFIMA models have been useful in fields as diverse as hydrology and economics (Hurst *e.t al*., 1965). Structural breaks are persistent and pronounced macroeconomic shifts in the data generating process. The break is unknown in most of the application but is possible to suspect that a break occurred sometime between two dates t_o and t_1 and there are classes of structural change tests that are in use to study such breaks which have been receiving much attention in both the statistics and econometrics communities (Zeileis, 2005).

2.0 Literature Review

Numerous methods exist for analyzing univariate time series such as autoregressive moving average, autoregressive integrated moving average, volatility models and so on (Hamilton, 1994). Lebo (1998) identified the importance of using ARFIMA specifications in bivariate and multivariate models. Cheung and Lai (1993) estimated ARFIMA models using oil price by relaxing the assumption that the residuals needed to be I(0), introducing the idea that the residuals could be fractionally integrated. Volatility spillover from the crude oil market to the three markets has been far greater than the spillover in the opposite direction, especially when oil prices are high [Ji and Fan (2012)].

Sibbertsen (2004) and Banerjee and Urga (2004) survey some of the issues associated with distinguishing long memory processes from some simple structural break models. Baillie and Bollerslev (1994) find the degree of persistence in the forward discount’s autocorrelations. They also estimated ARFIMA models and *d*, the order of fractional integration, equal to 0.45, 0.77 and 0.55 for Canada, Germany and the UK, respectively. Granger and Hyung(2004) and Hyung *et. al.* (2004) show that long memory behavior can be easily generated from structural breaks or regime switching. Maynard and

Phillips (2001) find evidence of slow decay in autocorrelation of forward discount and confirmed that the forward discount is dominated by a non-stationary long memory component with range $0.882 \le d < 1$ indicating that the forward discounts have non-stationary properties. Efe and Pinar (2013) examines the long-run relationships between the spot and future prices of Istanbul Stock Exchange 30 index (ISE-30) and foreign currencies including the Turkish Lira-US Dollar (TL/USD) and Turkish Lira-Euro (TL/EUR). Our research consider the West Texas Index (WTI), Brent crude oil prices, identified structural breaks, investigate long memory and discovered that ARFIMA model is the best to study and forecast oil price increase/decline.

3.0 Methodology

3.1 Data

The data for this research are the weekly crude oil price from two world major markets namely West Texas Intermediate (WTI) and Brent from 15/5/1987 to 20/12/2013. The data can be found in the web address http://tonto.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=PET&s=RWTC&f=W

3.2 Software

The statistical software used is R, Gretel and STATA version 12.

3.3 Long Memory Method

Approaches used for testing and estimating long memory parameters are the Local Whittle Estimator (LWE) and Maximum Likelihood ML estimation.

3.3.1 Local Whittle Estimator

The LWE is a semi parametric Hurst parameter estimator based on the periodogram. It was initially suggested by Kunsch (1987) and later developed by Robinson (1994). It assumes that the spectral density $f(\lambda)$ of the process can be approximated by the function

$$f_{C,H}(\lambda) = C\lambda^{1-2H} \tag{1}$$

for frequencies λ in a neighborhood of the origin.

The LWE of the Hurst parameter, $\widehat{H}_{LWE}(m)$, is implicitly defined by minimizing

$$\sum_{j=1}^{m} \log f_{C,H}(\lambda_{j,N}) + \frac{I_N(\lambda_{j,N})}{f_{C,H}(\lambda_{j,N})}$$

with respect to C and H, with $f_{C,H}$ defined in (1).

3.3.2 Geweke and Porter-Hudak Estimator

Under the null hypothesis, of no long memory (d = 0), the t-statistic

$$t_{d=0} = \widehat{d}\left(\frac{\pi^2}{6\sum_{j=1}^{g(n)} (U_j - \bar{U})^2} \right)^{-1/2} \tag{2}$$

where $U_j = \ln[4\sin^2(\omega_j / 2)]$ and $\bar{U}$ is the sample of U_j, $j = 1, \ldots, g(n)$.(see Geweke and Porter-Hudak, 1983)

3.4 The ARFIMA Model

The general form of an ARFIMA(p,d,q) model can be given as:

$$\phi(B)(1-B)^d X_t = \theta(B)\varepsilon_t \qquad \text{for} \qquad 0 < d < 0.5 \tag{3}$$

where the parameter d is a non-integer value, X_t is the dependent variable at time t, ε_t is distributed normally with mean 0 and variance σ^2, and $\phi(B)$ and $\theta(B)$ represent AR and MA components with lag operator *B*, respectively.

In a fractional model, the power is allowed to be fractional, with the meaning of the term identified using the following formal binomial series expansion

$$(1-B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k$$

$$= \sum_{k=0}^{\infty} \frac{\Pi_{a=0}^{k-1}(d-a)(-B)^k}{k!}$$

$$= 1 - dB + \frac{d(d-1)}{2!} B^2 - \cdots \tag{4}$$

3.6 ARCH-LM test

This test is used to check if the error, ε_t in the resulting model residuals is truly a skedastic function. The regression is given thus:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + u_t \tag{5}$$

where, $\alpha_1, \ldots, \alpha_p$ are the coefficients of the regression and α_0 is the intercept.

$H_o : \alpha_1 = \alpha_2 = \ldots = \alpha_p = 0$ There are no ARCH effects in the residuals under the null the LM statistic is distributed asymptotically as $\chi^2(p)$ statistic.

3.7 Single break/change test

A test to determine the existence of break, when the potential break at point t_o is known was introduced by Chow (1960) and modified to the Quandt Likelihood Ratio (QLR) test for break between t_0 and t_1. The QLR test statistic or the Supremum F statistic is given by:

$$Sup\, F = \max\{F(t_0), F(t_0+1), \ldots, F(t_1)\} \tag{6}$$

the Sup F statistic is the largest of many F statistics. The test rejects null hypothesis of no structural break if one of the computed F statistic gets larger than a certain critical values.

3.8 Multiple Structural Change/Break

Our study utilizes the generalized fluctuation tests (Gft), designed to bring out departure from constancy in a graphic way instead of expressing in terms of parameters particular type of departure in advance and then developing formal significance tests intended to have high power against F statistic approach

(Brown *et. al.*, 1975). It also provides the ability to identify as much as possible the numerous breaks that are present in a series.

3.9 Information Criterion

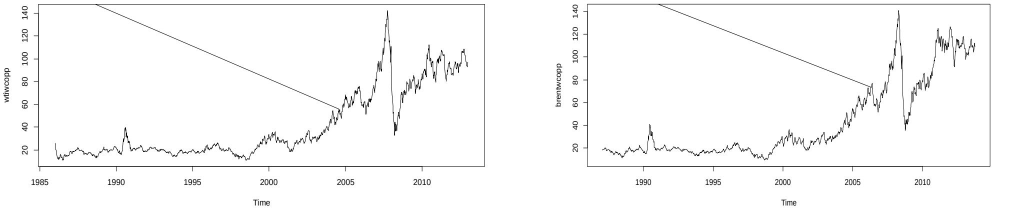
The Akaike information criterion (AIC) is a measure of the relative goodness of fit of a statistical model. Akaike (1974) suggests measuring the goodness of fit for some particular model by balancing the error of the fit against the number of parameters in the model. It provides the measure of information lost when a given model is used to describe reality. AIC values provide a means for model selection and cannot say anything about how well a model fits the data in an absolute sense. If the entire candidate models fit poorly, AIC will not give any warning of that.

The AIC is defined as

$$AIC = 2k - 2\ln(L) \tag{7}$$

where k is the number of parameters in the statistical model, and L is the maximized value of the likelihood function for the estimated model.

4.0 Empirical results



(a) (b)

Figure 1: Time plot of (a) WTI market and (b) Brent market weekly crude oil prices

The weekly crude oil price graph for both markets, the WTI and Brent are in Figure 1 (a) and (b) respectively. The two series indicates stable price and followed by a gradual increase. Furthermore, structural break/changes are clearly visible in both the overall series.

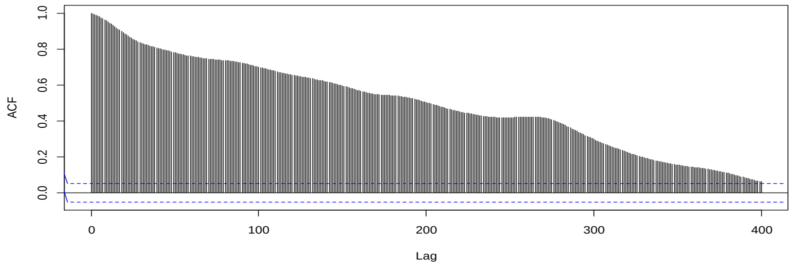


Figure 2: Plot of autocorrelation function for WTI series

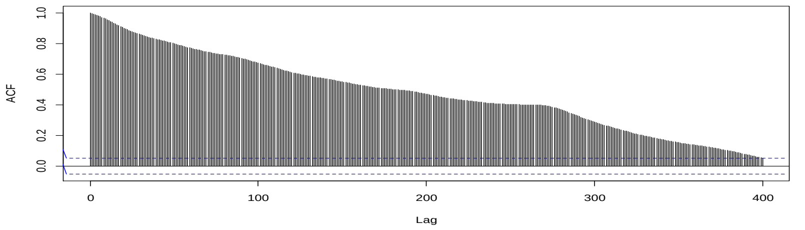


Figure 3: Plot of autocorrelation function for Brent series

The ACF of WTI and Brent price are displayed in Figure 2 and 3 above respectively and both experienced a slow decay which is a typical behaviour of a long memory process.

3.1 Unit root test.

Table 1: Unit root test for WTI and Brent series

	ADF t-statistic	**ADF-GLS**	**KPSS**	**Lag length**
WTI	-1.5673 (0.4994)	-3.0141 (0.1241)	2.1280 (0.148)	32
BRENT	1.0834 (0.7247)	-2.3751 (0.2322)	1.5311 (0.148)	35

Values in parenthesis are p-values.

The Augmented Dickey Fuller tests (ADF), the Generalized or modified Augmented Dickey Fuller test (ADF-GLS) and Kwiatkowski Phillips Schmidt and Shin (KPSS) unit root test of the two market prices are displayed in Table 1. Here, the most powerful unit root test among the three considered, the ADF-GLS (Stock, *et al.*, 1996), is consider for passing judgment about the stationarity and non-stationarity of the series. In the test, a constant and a trend is included in the regression and due to Ng and Perron (2001), the lag length of the test regression was selected by the modified AIC with a maximum lag of 35. The ADF-GLS unit root test provide mixed result for order of integration of the crude oil weekly prices while null hypothesis of unit root in the oil series is accepted. In addition, the results for KPSS and ADF test indicate the non-stationarity of the crude oil prices and further confirmed by the large p-value of the two tests.

4.2 Structural change/break test

Table 2 displays the preliminary test result of structural change using Quandt Likelihood Ratio (QLR) test for a break at all possible dates, t, say May 15, 1987 to Dec 20, 2013 while the OLS-CUSUM test whether the mean of the weakly crude oil price (WTI and Brent) did not change over the weeks. Both the null hypothesis for no structural breaks/change and constant price are rejected because the two statistic, often known as Sup.F and S_0 are too large and the p-values are less than 5% level of significance for both WTI and Brent series. The test based on F statistic each suggests that break occurred once in WTI market series in 2004 while it was observed in 2005 in Brent series.

Table 2: crude oil price structural change test

Test type	**WTI**	**BRENT**
QLR test	**662.782 (0.0000)**	**5526.812 (0.0000)**
OLS-CUSUM test	**16.2327 (0.0000)**	**15.6822 (0.0000)**
Number of break	**1**	**1**

Values in parenthesis are p-values.

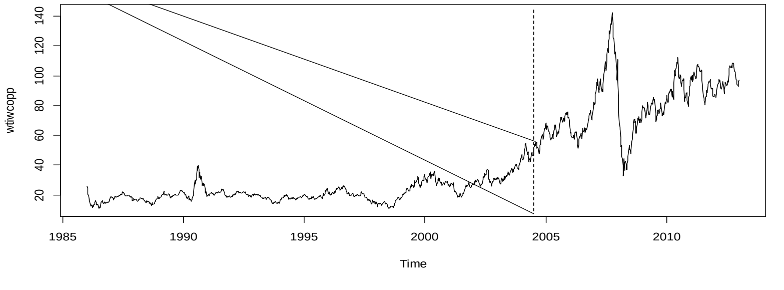


Figure 4: Graph of WTI series and the single break identified

The original WTI series with one structural break is displayed in Figure 4. The obvious reason for that structural break/change in WTI series may be attributed to the low spare capacity.

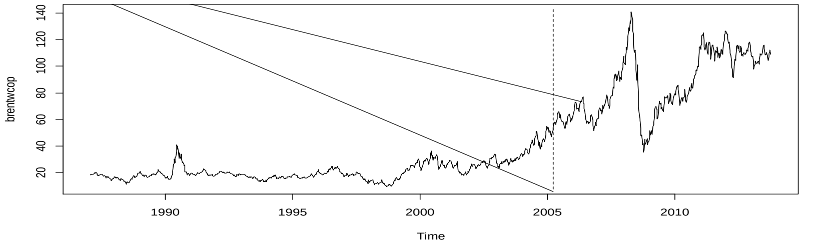


Figure 5: Graph of Brent series and the single break identified

Figure 5 above clearly shows the original series and the single structural break/change earlier discovered by both F statistic and empirical fluctuation process in the Brent series.

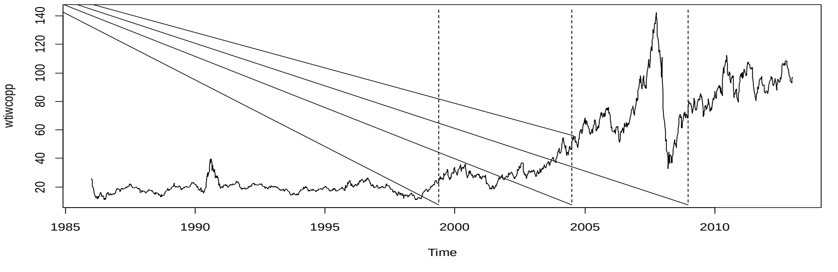


Figure 6: Plot of WTI series with the three the identified breaks

The three break for WTI are displayed in Figure 6 with the first, second and last break captured in 1999, 2004 and 2008 respectively which may be attributed to Asian financial crisis, OPEC cuts targets 1.7mmbpd, low spare capacity and global financial collapse respectively.

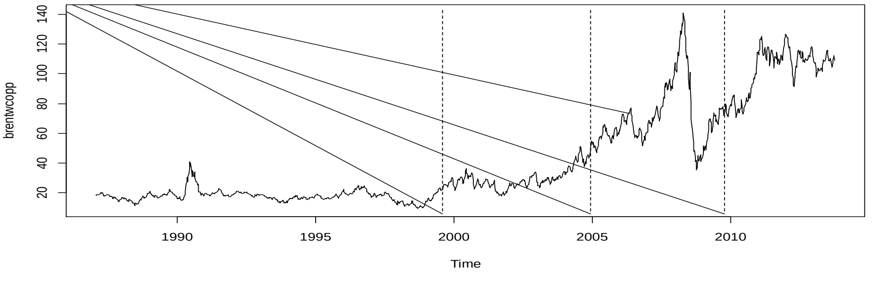


Figure 7: Plot of Brent series with the three breaks identified

The breaks for Brent are shown in Figure 7 and this pointed out that Brent series has break points in 1999, 2005 and 2009 which may also be attributed to OPEC cuts target 1.7mmbpd, 9-11 attacks, low spare capacity and OPEC cuts targets 4.2mmbpd. WTI is more expensive than Brent crude oil. Therefore, it is an alternative to WTI. So when there is an intervention that leads to structural break, the effect on WTI takes place almost immediately while the effect of the intervention is delayed by one period. This probably account for difference in the break point for WTI (1999, 2004 and 2008) and Brent crude (1999, 2005 and 2009).

4.3 Long memory parameter estimation

Table 3: Long memory estimate of WTI and Brent

DATA	LWE	GPH
WTI	1.11(0.0566)	0.65(0.0001)
BRENT	1.04(0.0577)	0.69(0.0002)

Values in parenthesis are p-values

Apart from the presence of long memory characteristics discovered after examination of autocorrelations plots of the series used in this study, the long memory formal test conducted using WTI and Brent data are displayed in Table 3. The LWE estimated d parameter to be 1.11 for WTI and 1.04 for Brent while Geweke and Porter-Hudak method based on Haslett and Raftery (1989) and Brockwell and Davis (1987, sec. 12.4), GPH provides fractional difference parameter approximately equal 0.7 based on the range of data used and these values lies within the conventional long memory parameter.

4.4 Model identification

Table 4: ARFIMA model candidates of WTI series

ARFIMA(p,d,q)	d	C.I for d (95%)	AIC
ARFIMA(0,d,0)	0.4999(0.0001)	0.4996,0.5001	478.15
ARFIMA(0,d,1)	0.4998(0.0003)	0.4992,0.5003	492.06
ARFIMA(1,d,0)	0.1456(0.0251)	0.0964,0.1949	501.88
ARFIMA(2,d,0)	0.0889(0.0360)	0.0185,0.1594	351.42*
ARFIMA(1,d,2)	0.4688(0.0469)	0.3769,0.5607	343.56*
ARFIMA(0,d,2)	0.4997(0.0005)	0.4988,0.5006	477.62
ARFIMA(0,d,3)	0.4995(0.0007)	0.4982,0.5009	452.18

Values in parenthesis are p-values

Table 5: ARFIMA model candidates of Brent series

ARFIMA(p,d,q)	d	C.I for d (95%)	AIC
ARFIMA(0,d,0)	0.4999(0.0001)	0.4997,0.5001	411.83
ARFIMA(2,d,0)	0.0918(0.0439)	0.0059,0.1779	398.94*
ARFIMA(0,d,1)	0.4998(0.0002)	0.4993,0.5003	399.71
ARFIMA(1,d,2)	0.4698(0.0472)	0.3773,0.5622	374.73*
ARFIMA(1,d,0)	0.1795(0.0267)	0.1272,0.2319	397.15
ARFIMA(0,d,3)	0.4996(0.0006)	0.4984,0.5007	401.43
ARFIMA(0,d,2)	0.4997(0.0004)	0.4989,0.5005	455.01

Values in parenthesis are p-values

ARFIMA models identification are in Tables 4 and 5, models with asterisk, *, are selected to be appropriate based on minimum information criteria. For both series(WTI and Brent), ARFIMA(2,d,0) and ARFIMA(1,d,2) model are suitable due to minimum AIC.

4.5 Model estimation

Table 6: ARFIMA model estimation using WTI series

Candidate Models	d	Constant	AR(1)	AR(2)	MA(1)	MA(2)
ARFIMA(2,d,0)	0.089(0.035)	46.1(16.630)	1.06(0.045)	-0.07(0.044)	-	-
ARFIMA(1,d,2)	0.469(0.047)	47.0(41.510)	0.95(0.013)	-	-0.27(0.048)	-7(0.029)

Values in parenthesis are p-values

Table 7: ARFIMA model estimation using Brent series

Candidate Models	d	Constant	AR(1)	AR(2)	MA(1)	MA(2)
ARFIMA(2,d,0)	0.092(0.043)	48.7(20.720)	1.11(0.053)	- 0.11(0.051)	-	-
ARFIMA(1,d,2)	0.469(0.047)	49.1(45.185)	0.95(0.014)	-	-0.23(0.048)	-0.15(0.031)

Values in parenthesis are p-values

Parameters of the models selected and estimated are displayed in Table 7 using WTI and Table 8 using Brent series and careful examination of the results revealed that the models are statistically significant.

4.6 Model diagnostic testing

Table 8: Residuals Normality and ARCH-LM test for WTI series candidate models

Candidate Models	AIC	Residuals Normality	ARCH-LM test
ARFIMA(2,0.89,0)	351.42	(0.97351,0.034)	176.340 (0.7641)
ARFIMA(1,0.47,2)	343.56	(0.2435,1.0162	287.142 (0.9857)

Normality has mean and variance of residuals in parenthesis while ARCH-LM test has p-values enclosed.

Table 9: Residuals Normality and ARCH-LM test for Brent series candidate models

Candidate Models	AIC	Residuals Normality	ARCH-LM test
ARFIMA(2,0.09,0)	398.94	(0.2435,1.0162)	291.482 (0.9914)
ARFIMA(1,0.50,2)	374.73	(0.9735,0.0364)	199.740 (0.6582)

Normality has mean and variance of residuals in parenthesis while ARCH-LM test has p-values enclosed.

The ARCH-LM and normality tests for residuals are displayed in Table 8 and 9. ARFIMA(1,0.47,2) and ARFIMA(2,0.09,0) model are good to study WTI and Brent series respectively because their residuals are similar to a white process and both models further shows evidence of no ARCH effect using ARCH-LM test.

4.7 Forecasting

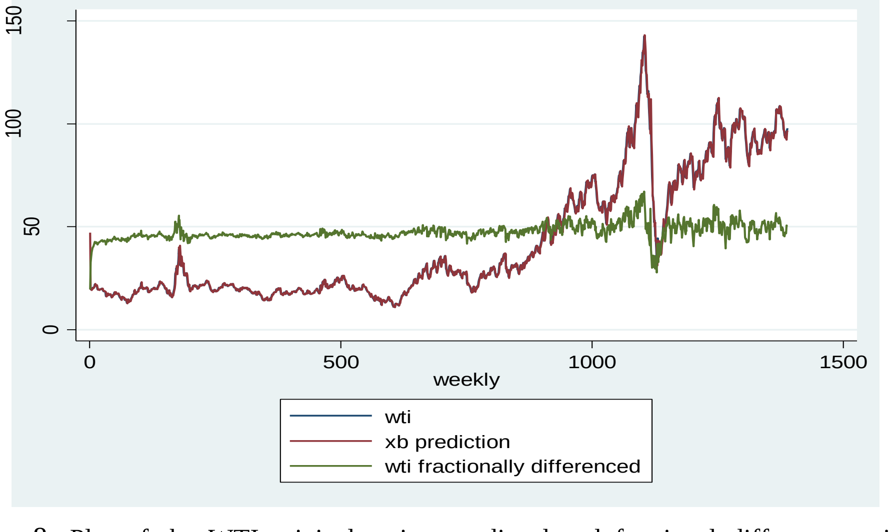


Figure 8: Plot of the WTI original series, predicted and fractional difference series of ARFIMA(1,0.47,2).

Table 10: WTI in sample one year forecast using ARFIMA(1,0.47,2) model.

S/No	Original Series (WTI)	In Sample Forecast(WTI)
1338	90.1	88.9
1339	92.8	91.5
1340	93.4	92.1
1341	94.6	93.3
1342	95.4	94.1
1343	97.3	96.1
1344	96.2	94.9
1345	97.0	95.7
1346	94.4	93.1
1347	92.2	90.9
1348	91.0	89.7
1349	92.7	91.4
1350	93.1	91.8
1351	96.1	94.8
1352	95.1	93.8
1353	93.4	92.1
1354	88.0	86.7
1355	91.0	89.7
1356	93.4	92.1
1357	95.8	94.6
1358	94.7	93.4
1359	94.8	93.5
1360	93.3	92.0
1361	94.3	93.0
1362	96.4	95.1
1363	96.7	95.4
1364	95.8	94.6
1365	100.7	99.4
1366	104.7	103.4
1367	106.9	105.6
1368	105.9	104.6
1369	105.5	104.3
1370	105.2	103.9
1371	107.0	105.7
1372	105.5	104.2
1373	108.3	107.1
1374	108.8	107.5
1375	108.4	107.1
1376	106.2	104.9
1377	103.1	101.8
1378	103.1	101.9
1379	102.7	101.4
1380	101.5	100.2
1381	97.6	96.3
1382	96.9	95.7
1383	94.3	93.0
1384	93.9	92.7
1385	93.9	92.6
1386	93.0	91.7
1387	96.2	94.9
1388	97.2	96.0
1389	97.9	96.6

After selecting ARFIMA(1,0.47,2) model for WTI series, Figure 8 displayed the original WTI series, predicted and fractional difference series. The part that contains the fractional difference series show that the WTI series is generated by a long memory processes with fractional differencing parameter d= 0.469 and that differencing the series using integrating order other than what followed the model will result in over differencing the series. Therefore the fractional difference WTI series have constant mean and stable variance thus stationary and is ready for time series modeling and forecasting using an ARFIMA model. To measure the reliability of the model selected, in sample forecast was carried out by reducing the WTI original observations by one year, perform the forecast and discovered that the marginal difference between the original series reduced and the in sample forecast result is small and the results are displayed in Table 10. The result of one year out sample forecast is in Table 11 and the result shows that in the near future, crude oil price may decline.

Table 11: WTI out sample one year forecast using ARFIMA(1,0.47,2) model.

Prediction(WTI)	Lower Bound (95%)	Upper Bound (95%)
96.5	91.6	101.4
96.3	90.2	102.5
96.1	88.8	103.4
95.8	87.5	104.2
95.6	86.2	104.9
95.3	85.0	105.6
95.0	83.8	106.2
94.7	82.7	106.8
94.4	81.5	107.3
94.1	80.4	107.8
93.8	79.4	108.3
93.5	78.4	108.7
93.2	77.4	109.1
92.9	76.4	109.4
92.6	75.4	109.7
92.3	74.5	110.0
92.0	73.6	110.3
91.7	72.8	110.6
91.4	71.9	110.8
91.1	71.1	111.0
90.8	70.3	111.2
90.5	69.6	111.4
90.2	68.8	111.6
89.9	68.1	111.7
89.6	67.4	111.9
89.4	66.7	112.0
89.1	66.1	112.1
88.8	65.4	112.2
88.6	64.8	112.3
88.3	64.2	112.4
88.0	63.6	112.5
87.8	63.0	112.5
87.5	62.5	112.6
87.3	61.9	112.6
87.0	61.4	112.7
86.8	60.9	112.7
86.6	60.4	112.7
86.3	59.9	112.8
86.1	59.4	112.8

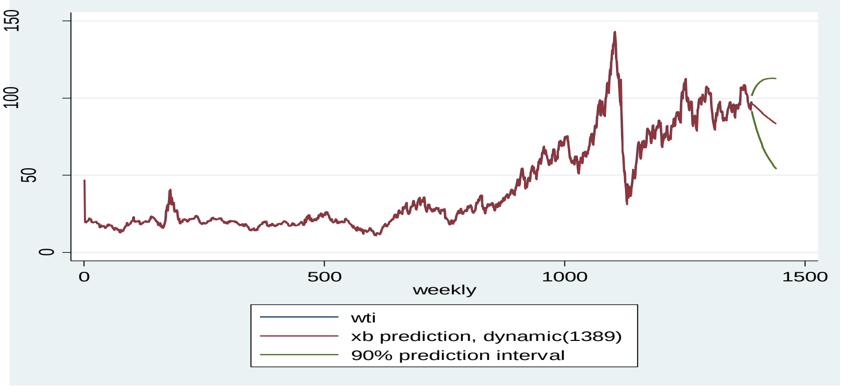


Figure 9: Plot of the WTI original series, predicted and one year forecast result of ARFIMA(1,0.47,2)

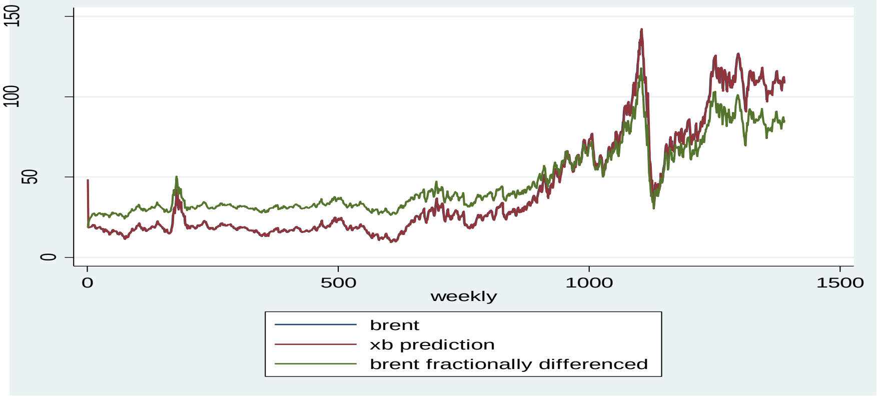


Figure 10: Plot of the Brent original series, predicted and fractionally difference series of ARFIMA(2,0.09,0)

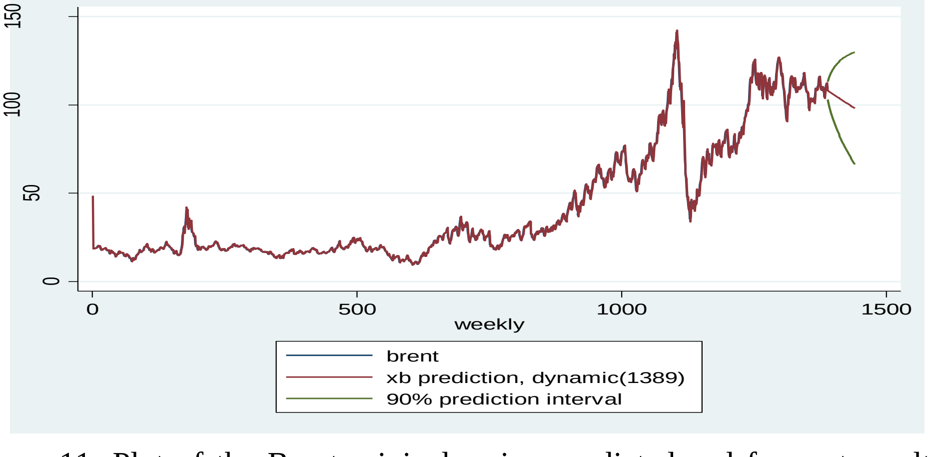


Figure 11: Plot of the Brent original series, predicted and forecast result of ARFIMA(2,0.09,0)

Table 12: Brent in sample one year forecast using ARFIMA(2,0.09,0) model.

S/No	Original Series (WTI)	In Sample Forecast(WTI)
1338	109.8	108.8
1339	112.4	111.4
1340	112.4	111.4
1341	111.4	110.4
1342	113.7	112.7
1343	115.1	114.1
1344	117.2	116.2
1345	118.1	117.1
1346	115.3	114.3
1347	112.4	111.4
1348	110.0	109.0
1349	108.4	107.4
1350	107.3	106.3
1351	107.7	106.7
1352	107.1	106.1
1353	103.3	102.3
1354	98.1	97.1
1355	100.6	99.6
1356	101.5	100.5
1357	103.8	102.8
1358	102.9	101.9
1359	102.3	101.3
1360	102.0	101.0
1361	102.9	101.9
1362	103.4	102.4
1363	103.9	102.9
1364	101.4	100.4
1365	105.2	104.2
1366	108.3	107.3
1367	109.4	108.4
1368	108.4	107.4
1369	108.6	107.6
1370	108.6	107.6
1371	110.7	109.7
1372	111.1	110.1
1373	115.3	114.3
1374	116.0	115.0
1375	113.2	112.2
1376	110.0	109.0
1377	108.8	107.8
1378	108.6	107.6
1379	110.3	109.3
1380	110.1	109.1
1381	107.8	106.8
1382	107.6	106.6
1383	104.5	103.6
1384	107.1	106.1
1385	109.3	108.3
1386	111.3	110.3
1387	112.3	111.3
1388	109.1	108.1
1389	110.3	109.4

Figure 10 shows the original Brent series and fractional difference result using the selected ARFIMA(2,d,0) model with d=0.0918. The in sample and out sample forecast are in Table 12 and 13 respectively. The in sample forecast resemble the original series, the out sample forecast indicate a steady decline and is displayed in right hand side of Figure 11.

Table 13: Brent out sample one year forecast using ARFIMA(2,0.09,0) model.

Prediction(WTI)	Lower Bound (95%)	Upper Bound (95%)
108.1	103.2	113.0
107.9	101.5	114.2
107.7	100.1	115.2
107.4	98.8	116.1
107.2	97.6	116.9
107.0	96.5	117.6
106.8	95.4	118.3
106.6	94.4	118.9
106.4	93.4	119.4
106.2	92.5	120.0
106.0	91.6	120.5
105.8	90.7	120.9
105.6	89.9	121.4
105.4	89.1	121.8
105.2	88.3	122.2
105.0	87.5	122.6
104.8	86.7	123.0
104.6	86.0	123.3
104.5	85.2	123.7
104.3	84.5	124.0
104.1	83.8	124.3
103.9	83.1	124.6
103.7	82.4	124.9
103.5	81.8	125.1
103.3	81.1	125.4
103.1	80.5	125.7
102.9	79.8	125.9
102.7	79.2	126.1
102.5	78.6	126.4
102.3	78.0	126.6
102.1	77.4	126.8
101.9	76.8	127.0
101.7	76.2	127.2
101.5	75.6	127.4
101.3	75.1	127.6
101.1	74.5	127.8
100.9	73.9	127.9
100.7	73.4	128.1
100.5	72.8	128.3
100.4	72.3	128.4
100.2	71.8	128.6
100.0	71.2	128.7
99.8	70.7	128.8
99.6	70.2	129.0
99.4	69.7	129.1
99.2	69.2	129.2
99.0	68.7	129.4
98.8	68.2	129.5
98.7	67.7	129.6
98.5	67.2	129.7
98.3	66.8	129.8
98.1	66.3	129.9

5.0 Conclusion

The research used an Auto Regressive Fractional Integrated Moving Average (ARFIMA) models to study weekly crude oil prices using WTI and Brent series for the period 15/5/1987 to 20/12/2013. Units root tests show that the two series are not stationary, long memory characteristics dominated the two prices and fractional differencing is the appropriate procedure to stabilize the observed variability. Several ARFIMA models were identified, estimated and diagnostic check of the selected models was carried out. ARFIMA(1,0.47,2) and ARFIMA(2,0.09,0) are the appropriate and best models to study WTI and Brent series respectively because their residuals are similar to a white process and both models residuals shows absence of ARCH effect using ARCH-LM test. In sample and out sample forecast was carried out using the recommended ARFIMA models. The in sample forecast for both series indicates that the difference between the original series and the in sample forecast is small or marginal and these further shows the reliability of the model estimated. The out sample forecast revealed a decline in the two oil prices. Hence, diversification of economy is necessary and recommend so that other sector of the economy such as agriculture, education, mining and so on could be given adequate attention in other to increase revenue generation and reduce over dependant on oil resources.

6.0 References

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