

## Estimating Bull and Bear Betas for the Nigerian Stock Market Using Logistic Smooth Threshold Model<sup>1</sup>

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*In this paper, we examine the Nigerian stock market sector returns and estimate the bull and bear betas using the Logistic Smooth Threshold Market (LSTM) model. The LSTM model specification follows from the linear Constant Risk Market (CRM) model. We estimate the LSTM model for the overall sampled daily time series from 2001 to 2012 using the conditional nonlinear least squares approach. We also estimate the model for each of the All share Index (ASI) sub-samples taking the time of financial crisis (February 2008) as the break point. The results show the significant correlations of stocks returns in each market industry with ASI. Nonlinear LSTM dynamics are found to be significant, with significant bull and bear betas in the overall and each of the sub-samples. We find in particular, that the Petroleum, Finance, and Food and Beverages sector equities to be of higher investment risk within the study period.*

**Key Words:** Logistic smooth threshold model, nonlinear least squares, market beta, market returns

**JEL Classification:** C22, C58

### 1.0 Introduction

Central banks are becoming more and more concerned with the functioning of financial markets because of their importance not only for monetary policy, but also for the effective regulation of the financial institutions regarding risk management. A critical component of financial markets is the capital market. The capital market provides a framework within which medium to long term resources are made available for productive utilization. In exchange for financial assets, lenders provide funds offered by borrowers. Like in any other market, investors in the Nigerian capital market are interested in appraising their investments and seek to know the level of risks associated across the business sectors. From the regulatory perspective, regulatory agencies were concerned about the causes and remedies for the downturn in investor confidence. As a matter of fact, the stability of the capital market means a lot

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to the transmission mechanism of monetary policy to the real sector. That is why some central banks publish the market Beta index for the “Banks, Finance and Insurance (BFI) Sector” in regular financial stability reports. A high BFI Beta index indicates an increase in stress levels.

Historically, the Nigerian stock market was established in 1960 but commenced active trading on June 5, 1961 first as the Lagos Stock Exchange but later (in 1977) renamed the Nigerian Stock Exchange (NSE). It commenced with few stocks: the Nigerian Tobacco Company, the Nigerian Cement Limited and two registered stocks for John Holt Investment Company Limited. By the end of December 2012, there were one hundred and ninety six (196) equities being traded. The exchange publishes the All Share Index (ASI) with January 3, 1984 as the base date and computed a Laspyres Index. In May, 2001, the index crossed the 10,000 mark and it rose to 10,153.8 at the end of the same month while the highest value of 66,371.20 was recorded on March 5, 2008. The market capitalization this date was ₦12.64 trillion. Like other stock markets, the Nigerian Stock Market was not spared by the global financial meltdown which saw the ASI recording 28,078.81 at the end of the year 2012 with a corresponding market capitalization of ₦8.97 trillion.

This study investigates the risk/return characteristics of the equities of the different business sectors traded on the Nigerian stock exchange with a view to assessing their relative risk levels using innovative way of incorporating regime-switching mechanism in the traditional Capital Asset Pricing Model (CAPM). The issue of stationarity of the beta in the Nigerian stock market over the bulls and bears periods forms the crux of this study. Our specific objective therefore is, to estimate the bull and bear betas for equities of key economic sectors. Because the bull/bear state is not observable, many of the existing studies on the Nigeria stock market assume that the Beta is constant over the two market regimes. Using realized and expected returns, a few other studies have employed the Dual Beta Models (DBM) to look at the market.

The remainder of this paper is structured as follows: section 2 reviews past literatures on the subject matter. Section 3 presents the methodologies involved in the research, while section 4 presents the data analysis and discusses the results as well. Section 5 gives the concluding remarks and policy implications of the findings.

## 2.0 Literature Review

Investors are usually motivated by expected returns to invest in stocks (Gitman and Joehnk, 1996), while watching both diversifiable and non-diversifiable risks. The CAPM looks at returns as a function of the level of non-diversifiable risk investments are exposed to. The CAPM, which was developed independently by many writers such as Sharpe (1964) and Litner (1965), marked the birth of asset pricing theory.

Despite numerous theoretical and empirical criticisms, the CAPM has been and is still one of the most popular standard tools for financial research. It is an extension of Markowitz's portfolio theory of portfolio returns and expected risk, with the Beta being the coefficient when expected return is regressed on market risk. However, on the contrary, Ferson and Korajczyk (1995) and Jaganathan and Wang (1996) argued that beta and the market risk premium change with time and are not static as proposed by the CAPM. They suggested that the CAPM should be adjusted to incorporate time variation element in its computation of asset prices.

The Arbitrage Pricing Model (APM) is an alternative to the CAPM. The APM sees the return on assets as a function of several risk factors. It assumes that investors take "advantage of arbitrage opportunities in the broader market" with an asset's rate of return being a function of the return on alternative investments and other risk factors (Ferson and Harvey, 1998). In addition, other studies like Kandir (2008) found prices like exchange, interest and inflation rates, affecting portfolio returns.

Over the bulls and bears market segments, studies like Fabozzi and Francis (1977) and Kim and Zumwalt (1979) have found market betas to significantly differ. The bulls and bears market segments are widely used to characterize the evolution in stock prices over time. These segments are often not observable, thereby informing the consideration of the state definition in terms of returns (Fabozzi and Francis, 1977; Kim and Zumwalt, 1979 and Chen, 1982). The "bull" and "bear" market structures can also be termed as periods of expansion and contraction in the business setting. Thus, the two market phases are associated with periods when the returns are positive and negative (Aslanidis *et al.*, 2002).

The Smooth Transition Autoregressive (STAR) model typically describes the financial market classified into two phases. The market index as an example

displays critical threshold value which separates the "bull" from the "bear" market periods (Wiggins, 1992). Granger and Silvapulle (2001) separate the market into the "bullish" and "bearish" periods. These "bull" and "bear" phases are defined based on peaks and troughs found in economic and financial data. So, the "bull" and "bear" markets are defined in terms of movements between peaks and troughs. Research has shown that "bull" markets are said to last longer than "bear" markets (Lunde and Timmermann, 2001; Pagan and Sossounov, 2003). In stock market, the "bull" and "bear" markets correspond to periods of generally increasing and decreasing market prices.

Following Hamilton (1989) analysis of the bull and bear markets, Schwert (1989), Hamilton and Susmel (1994), Turner *et al.* (1989), Ang and Bekaert (2002) and Guidolin and Timmermann (2004) adopted the framework to study changes in volatility and regime switching between bull and bear markets. Quite a number of researchers have investigated the relationship between beta risk and stock market conditions. While, Fabozzi and Francis (1977,1979), Chen (1982), Dukes *et al.* (1987) and Wiggins (1992), looked at changes in returns, Bhardwaj and Brooks (1993) and Granger and Silvapulle (2001) looked at median and quantiles of returns in looking at bulls and bears segments.

In defining the bull and bear markets, Cohen *et al.* (1973, 1987) used changes of at least 20% from trough to peak in the S&P500 index. After Duke *et al.* (1987), Pagan and Sossounov (2003) and Lunde and Timmermann (2001), proposed algorithms to classify bulls and bears markets. The bull and bear markets regimes were also defined by Gonzalez, *et al.* (2005) in a regime-switching model. The Pagan and Sossounov (2003) algorithm was utilized by Cunado *et al.* (2008) and Gursakal (2010) to classify the S&P500 index into bull and bear phases. In a more recent study, Gil-Alana *et al.* (2014) also applied the Pagan and Sossounov (2003) algorithm to stocks in Europe, America and Asia using the 20%'s rule to classify stocks into phases and obtained results to Cunado *et al.* (2008).

In addition to the Pagan and Sossounov (2003) algorithm, regime switching models for classification of bull and bear markets are being used. These include the Markov switching in Maheu and McCurdy (2000), continuous

switching proposed by Granger and Teräsvirta (1993) and Teräsvirta (1994), and the nonlinear market model of Woodward and Anderson (2009).

In Nigeria, a number of studies have been conducted on the estimation of the stock market beta, most of which used the ordinary least squares (OLS) method with static beta. Some of the investigations include; Oludoyi (2003) who examined risk characteristics of quoted firms, and Akingunola (2006) studied the CAPM for Nigerian stocks. Bello and Adedokun (2011) also studied the beta of stocks of Nigerian firms.

Olakojo and Ajide (2010) examined the CAPM for the Nigerian stock market using monthly stock returns for 10 most capitalized stocks on the exchange, while Osamwonyi and Asein (2012) examined the market risk as defined in the CAPM as an explanatory variable for security returns. Their findings do not support the theory's basic statement that "higher beta is associated with higher returns" and thus concluded that the CAPM does not hold for Nigeria. In other words, they found that the value-beta relationship was non-linear but failed to model the relationship in a non-linear manner, as they used OLS method with a constant beta.

This study will employ the asset pricing model as its theoretical framework where the ASI and business sector indexes of the Nigerian stock exchange would be used to represent the asset price. This will be of immense benefit to investors, researchers and policy makers.

### **3.0 Methodology**

The workhorse model for an analysis of this nature is the traditional CAPM, which posits that in a well-diversified portfolio of assets, the valuation of a security depends not only on its own returns, but on how it contributes to overall risk. The beta coefficient measures the relation between returns on a particular security and returns on the overall market portfolio. The CAPM sees risky stocks as having higher betas and discounted at high rates, while less sensitive stocks have lower beta and discounted low rates.

In this work, the logistic smooth transition market model (LSTM) is the preferred model since it measures the speed of transition between the market phases and as well classifies the market into the two distinguished phases. It is applied to the indices of the different sectors of the stock market over the period 2001 to 2012 to investigate whether bull and bear market betas differ.

The DBM model implies discrete jump regimes, while the LSTM allows for a smooth and continuous transition between bull and bear states (i.e.). This is based on our believe that in stock In markets with many participants, smooth transition between bull and bear seems more appropriate due to heterogeneous beliefs and differing investment horizons.

### 3.1 Logistic Smooth Threshold Model (LSTM)

Following a constant risk CAPM, an unconditional beta for an asset or portfolio can be estimated based on the regression:

$$R_{it} = \alpha + \beta R_{mt} + \varepsilon_{it} \tag{1}$$

where  $R_{it}$  is the return on asset or portfolio  $i$ , for period  $t$ ,  $R_{mt}$  is the return on the market index for period  $t$ , and  $\varepsilon_{it}$  is disturbance term which is assumed to follow a white noise process. The coefficient  $\beta$ , is the market risk for the asset/portfolio in question and is computed as  $Cov(R_{it}, R_{mt})/\sigma_{mt}^2$ . The formulation in equation (1) assumes that  $\alpha$  and  $\beta$  are constants over time. The model is therefore known as Constant Risk Model (CRM). In view of arguments in respect of the stability of  $\beta$ , a dual beta market (DBM) model introduces a threshold dummy into (1) which defines the bull and bear periods in the market as follows:

$$R_{it} = \alpha + \beta R_{mt} + \beta^U \cdot D_t R_{mt} + \varepsilon_{it} \tag{2}$$

The discrete dummy ‘ $D$ ’ takes a value 1 if the return on the market index exceeds a certain threshold,  $K$ , and zero otherwise. The parameter  $\beta^U$  is for the “up” or bull market. Our study, like several other studies, argues that the definition of the threshold level,  $K$ , used in (2) may be faulty and prefers to model the threshold level alongside the other parameters of the model. Again, the transition between the bull and bear periods is believed to be smooth, rather than discrete as specified in (2). Hence, the choice of the Logistic Smooth Threshold Model (LSTM), which is specified below to account for possible smooth and gradual transitions between the bull and bear periods:

$$R_{it} = \alpha + \beta R_{mt} + (\alpha^U + \beta^U \cdot R_{mt})G(S_t, \gamma, K) + \varepsilon_{it} \tag{3}$$

where  $G(S_t, \gamma, K)$  is the transition function, normalized and bounded between 0 and 1,  $S_t$  is the threshold or transition variable (which is  $R_{mt}$  in this case),  $\gamma$  is the speed of transition (or the smoothness parameter),  $K$  is the threshold parameter and  $\varepsilon_t$  is the disturbance term, such that  $\varepsilon_t \sim (0, \sigma^2)$ . When  $\gamma$  is large, the shape of the transition function  $G(\cdot)$  is very steep in the neighborhood of the threshold value ‘ $K$ ’. We choose to use the logistic specification for the transition function as follows:

$$G(S_t, \gamma, K) = (1 + \exp[-\gamma(R_{mt} - K)])^{-1}, \gamma > 0 \tag{4}$$

As noted earlier, the LSTM model allows for gradual changes in both the level and trend of the market return series. The LSTM as specified in (2) and (3) classifies the market into a ‘bull’ regime when  $S_t > K$  and a bear regime when  $S_t < K$ . The incorporation of equation (3) in (2) allows beta to change monotonically with the transition variable  $S_t$  (or the independent variable  $R_{mt}$ ) due to the fact that  $G(\cdot)$  in (3) is a smooth and continuous increasing function of  $S_t$ . The transition function  $G(\cdot)$  takes a value between 0 and 1, depending on the magnitude of  $(S_t - K)$ . When  $(S_t - K)$  is large and negative,  $G(\cdot) = 0$  and  $R_{it}$  is effectively generated by the linear model in (1). In such cases, the market for stocks in industry  $i$  is in the bear state. On the other hand, when  $(S_t - K)$  is large and positive,  $G(\cdot) = 1$  and  $R_{it}$  is effectively generated by

$$R_{it} = (\alpha + \alpha^U) + (\beta + \beta^U) R_{mt} + \varepsilon_{it} \tag{5}$$

and the model for stocks in industry  $i$  is in the bull state. The parameter  $\beta^U$  in equation (5) measures the difference between the ‘bull’ and ‘bear’ market values of the slope coefficient. Thus, the bull beta is computed as  $\beta + \beta^U$  while the bear beta is represented as  $\beta$ . Intermediate values of  $(S_t - K)$  give a convex combination of the two extreme regimes, and as  $G(S_t - K)$  whereas from 0 to 1, the market for industry  $i$  moves through a series of market states that range from very bearish to very bullish.

The parameter  $\gamma$  determines the speed of transition between the two market states. As  $\gamma$  tends to 0, the LSTM model in (3) approaches the linear model in (1). These are the parameters of interest in this study and they are used to measure the market risk for each of the sectors of the Nigerian stock market during the bull and bear regimes. Due to the fact that the LSTM model presented in this work closely resemble LSTAR model, tests for smooth transition regression employed here are very similar. Using the LSTM to



study bull and bear market is a very new methodology. This methodology is straight forward and parametric approach is involved.

### 3.2 Model Identification and Specification

Granger and Teräsvirta (1993) strongly proposed a “specific to general” strategy for building nonlinear time series models. This says the specification of model for asset returns should start with a simple or restricted model to be proceeded by more complicated ones except if a model fails diagnostic tests, which indicate model inadequacy. Modeling cycle for LSTAR model adapted for LSTM model here as put forward by Teräsvirta (1994) consists of the following steps: specification of a linear CRM model of order  $p$  for the return series which is of order  $p = 1$ ; selecting the appropriate transition variable,  $S_t$ , and the form of the transition function which is logistic; testing of the null hypothesis of CRM linearity against the alternative of LSTM nonlinearity, and if linearity is rejected, proceed to estimate the LSTM parameters and evaluate the LSTM model using the diagnostic tests.

We apply a general-to-specific procedure, with the least significant (if nothing is significant) variable is dropped at each stage and the reduced model re-estimated. Teräsvirta (1994) suggests the use of Akaike Criterion (AIC), Bayesian Information Criterion (BIC) and Ljung-Box statistic to determine lag order of the model. The selected model by the general-to-specific procedure is then assumed to form the null hypothesis for testing linearity. Once the initial linear model is specified, we proceed to testing linearity against the LSTM form. The initial linear market model (CRM) is first estimated. Then, null hypothesis of linearity against LSTM is tested based on the hypothesis,  $H_0 : \gamma_i = 0$ . If the linearity hypothesis cannot be rejected, we conclude the CRM model adequately represents the data generating process, on the contrary, we can go on to estimate the nonlinear LSTM using Nonlinear Least Squares (NLS) method.

Luukkonen *et al.* (1988) concludes that tests  $H_0 : \gamma_i = 0$ , are not standard since the parameters of LSTM in (3) are only identified under the alternative hypothesis,  $H_1 : \gamma_i \neq 0$ . Then,  $G(\cdot)$  is then replaced by third-order Taylor series expansion, and expanding this gives the auxiliary regression model,



$$R_{it} = \phi_1 + \phi_2 R_{mt} + \phi_3 S_t + \phi_4 (S_t)^2 + \phi_5 (S_t)^3 + \phi_6 R_{mt} S_t + \phi_7 R_{mt} (S_t)^2 + \phi_8 R_{mt} (S_t)^3 + U_{it} \tag{6}$$

with the last six variables in the equation acting as proxies for the nonlinearity with  $H_0 : \phi_i (i=1, \dots, 8)$  as parameters in the model and  $U_{it}$  is some noise process. Then, testing  $H_0 : \gamma_i = 0$  against  $H_0 : \gamma_i \neq 0$  implies testing  $H_0 : \phi_j = 0 (j=3, \dots, 8)$  against  $H_1 : \text{at least one of } \phi_j (j=3, \dots, 8) \text{ is not zero}$ .

The test statistic has a  $\chi^2$  distribution with 6 degrees of freedom asymptotically and the test statistic as denoted by LM2. The test statistic is then computed based on it corresponding auxiliary regression as

$$LM2 = \frac{N(SSR_0 - SSR_1)}{SSR_0} \tag{7}$$

The F version of LM2 is then given as

$$F = \frac{(SSR_0 - SSR_1) / 6}{SSR_0 / (N - 7)} \tag{8}$$

where  $N$  is the sample size and  $SSR_0$  and  $SSR_1$  are the error sums of squares of the CRM in (1) and auxiliary model in (6), respectively. The acceptance of the linearity hypothesis,  $H_0 : \gamma = 0$ , indicate that two regime parameter is not possible.

### 3.3 Parameter Estimation

The estimation principle can be performed using any conventional nonlinear optimization procedure (Quandt, 1983; Hamilton, 1994) by an appropriate choice of starting and the estimation of the nonlinear parameter in the transition function. As suggested by Leybourne *et al.* (1998), the estimation procedure can be simplified by concentrating the sum of squares function.

The LSTM is estimated using Nonlinear Least Squares (NLS), and consistent estimates are obtained with the assumption that the errors,  $\varepsilon_t$  are iid(0,  $\sigma^2$ ). Using normality assumption, NLS is seen to be equivalent to MLE. Because

of the nonlinear component, estimating the parameters of the LSTM poses some difficulty, which often gives flat likelihood with respect to the parameters  $\gamma$  and  $K$  (Teräsvirta, 1994; Maringer and Meyer, 2008; Chan and Theoharakis, 2009). Failure of convergence is also experience during estimation. The estimates of  $\alpha, \hat{\alpha}^U, \beta$  and  $\hat{\beta}^U$  are sequentially conditioned on each value of  $\gamma$  and  $K$  as given by the Ordinary Least Squares (OLS) estimator to overcome these problems,

$$(\hat{\alpha}, \hat{\alpha}^U, \hat{\beta}, \hat{\beta}^U)' = \frac{\sum_{t=1}^N (\gamma, K) r_t}{\sum_{t=1}^N r_t (\gamma, K) r_t (\gamma, K)'} \quad (9)$$

The procedure for setting out a grid search over likely values for  $(\gamma, K)$  to determine the set  $(\hat{\gamma}, \hat{K})$  that minimizes the residual sum of squares is given by Maringer and Meyer (2008).

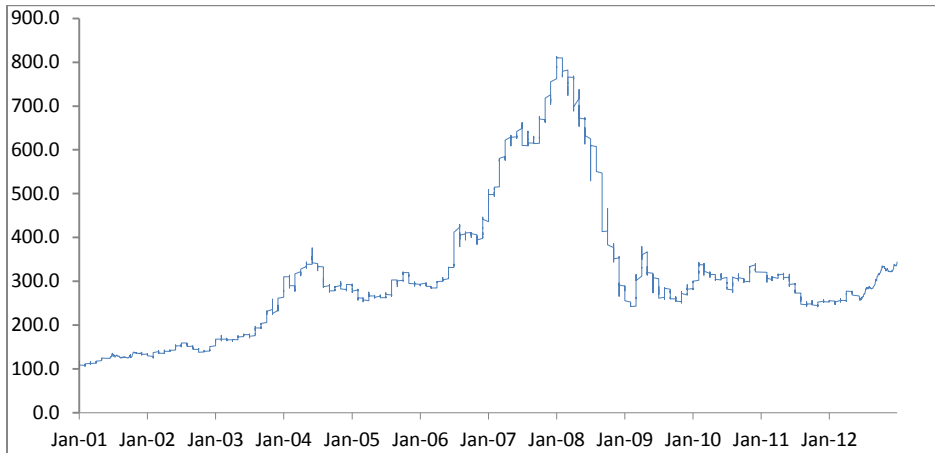
### 3.4 Research Data

The data used in this study are the daily stock prices of all listed stocks on the Nigerian Stock Exchange (NSE). The data span from 2<sup>nd</sup> January 2001 to 28<sup>th</sup> December 2012 giving rise to about 2967 data points. The All Share Index (ASI) used are as compiled and published by the NSE rescaled to have 2001 as the origin, while portfolio/sector indexes were computed by the researchers in accordance with the NSE-ASI computation methodology. Altogether, 15 Portfolios/Sectors are represented based on availability of data as well as importance of such in the Nigerian market. In computing sector indexes, unless where exact dates of changes in issued shares are reported, published end period (end December) values of issued share were used. No adjustments were made for non-trading days (weekends and holidays).

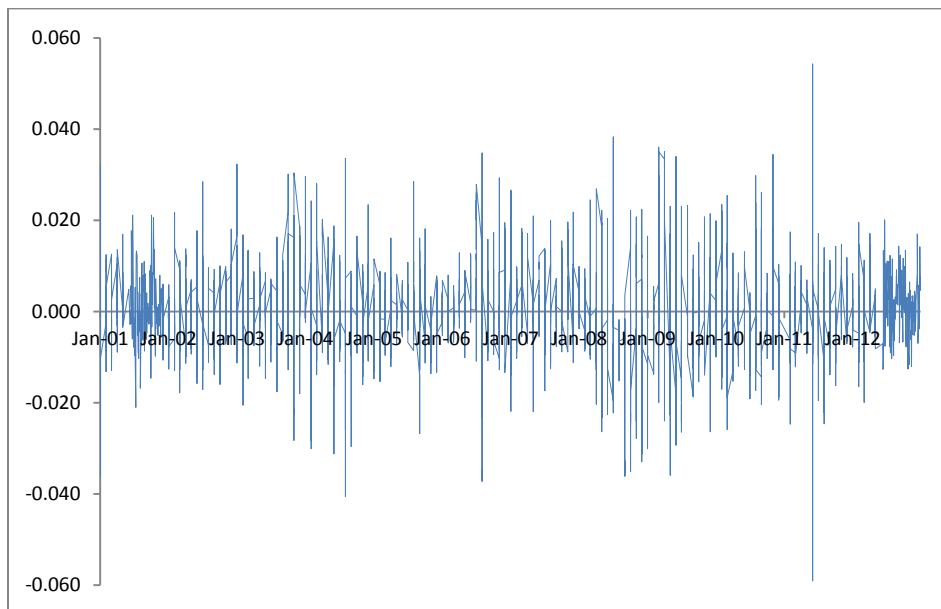
### 4.0 Results and Discussions

The Market Portfolios/Sectors examined are: Agriculture, Transportation, Finance, Food, Construction, Engineering and Technology, Footwear, Health, Industrial Products, Building, Conglomerate, Packaging, Petroleum, Printing and Chemicals. Return series for each Market portfolio/sector and the ASI are

computed as logarithmic returns. Figure 1a and 1b provide a graph of the ASI and logarithm returns from 2001 to 2012. We can observe that the global financial crisis took its toll on the Nigerian capital market in February 2008, only to stabilize a year later. Table 1 gives the descriptive statistics for the market returns of each of the 15 sectors.



**Fig. 1a:** Nigerian Stocks from 2001 to 2012



**Fig. 1b:** Log>Returns of ASI from 2001 to 2012

Turning to the summary statistics in Table 1, all sector logarithmic returns exhibited high volatility with the Jarque-Bera tests indicating that none of the return series followed a normal distribution. The Transportation, Finance,

Engineering and Technology, Health and Petroleum sectors are characterized by dominant positive returns as indicated by high positive skewness coefficients, while Agriculture, Printing and Chemical sector equity returns were characterized by dominant negative returns as indicated by high negative skewness coefficient.

**Table 1: Summary Statistics and Estimates of Beta for CRM**

Sectors	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Beta Estimate
Agriculture	0.0216	-3.7898	85.7283	852899.8***	0.1649***
Transportation	0.0705	24.9617	1122.0060	15500000.0***	0.1029**
Finance	0.0276	8.6830	178.3385	3836667.0***	0.2842***
Food	0.0277	2.0138	170.2166	3457567.0***	0.3324***
Construction	0.0358	3.3482	225.6918	6134240.0***	0.1577**
Engineering & Technology	0.0197	27.4518	1165.5710	16700000.0***	0.0769**
Footwear	0.0114	-0.6968	61.6378	425167.3***	-0.0119
Health	0.0236	4.3235	262.9478	8360121.0***	0.0667
Industrial Products	0.0219	2.9341	80.9296	754779.6***	0.1176***
Building	0.0075	0.3034	54.6970	330331.9***	0.0249
Conglomerate	0.0175	0.4947	176.1885	3706909.0***	0.1187***
Packaging	0.0136	-1.3147	35.2782	129613.8***	0.0786***
Petroleum	0.0262	5.8577	264.0176	8436720.0***	0.2347***
Printing	0.0507	-5.4916	660.5599	53450487.0***	-0.0199
Chemicals	0.0201	-2.8869	197.3667	4672904.0***	0.0912**
ASI	0.0098	-0.0287	5.4809	761.0***	

\*\*\* Significant at 1%, \*\* significant at 5%

#### 4.1 The CPM

Results of fitting the Constant Risk Market (CRM) given in (1) are given in the last column of Table 1. The CRM estimate of the model beta is highest for Food, followed by Finance and Petroleum sectors stocks, and this is lowest for Footwear sector. Three sectors, Printing, Building and Footwear, recorded insignificant CRM model beta estimates. This implies that 12 out of 15 CRM betas are significant at 5% level of significance, and this is in agreement with previous authors (Woodward and Anderson, 2009).

## 4.2 The LSTM

Evidence of nonlinearities of LSTM against CRM model is presented in Table 2. As discussed in the methodology, the approach employed is that of Luukkonen, *et al.* (1988) and Teräsvirta (1994). The estimates of sum of squares for CRM and nonlinearity test auxiliary regression model in (6) with the corresponding F-statistics are given. Based on the critical point set for the test, linearity between sectorial stock and overall ASI is rejected across all the sectors. Though there is a choice between exponential and logistic threshold market model, but since LSTM relates to bear and bull market, we assume that the nonlinear market model is LSTM for all the sectorial stocks.

**Table 2:** Linearity Tests Statistics

Sectors	SSR <sub>0</sub>	SSR <sub>1</sub>	F	Decision
Agriculture	1.3753	1.3439	11.26729**	LSTM
Transportation	14.7402	12.82297	64.18861**	LSTM
Finance	2.2422	2.1618	17.69575**	LSTM
Food	2.2458	2.2130	7.207587**	LSTM
Construction	3.7887	3.7613	3.569008**	LSTM
Engineering & Technology	1.1440	1.1069	16.00424**	LSTM
Footwear	0.3839	0.3496	44.09234**	LSTM
Health	1.6514	1.6299	6.425003**	LSTM
Industrial Products	1.4173	1.3956	7.555881**	LSTM
Building	0.1645	0.1630	4.5**	LSTM
Conglomerate	0.9070	0.8890	9.793826**	LSTM
Packaging	0.5489	0.4768	64.82301**	LSTM
Petroleum	2.0244	1.9955	7.045124**	LSTM
Printing	7.6098	7.5626	3.060948**	LSTM
Chemicals	1.1944	1.1523	17.3948**	LSTM

\*\*\* Significant at 1%, \*\* significant at 5%, \* significant at 10%.

Estimation of LSTM required the determination of initial values of  $\gamma$  and  $K$ . Their values were first determined by grid-search following the approach of Maringer and Meyer (2008). Though, this is a very difficult task, nonlinear optimization of most software report convergence problem during estimation, therefore the better method is the grid searching. The selected values of  $\gamma$  and  $K$  are the values that minimized the error in the LSTM transition function. We standardized the transition variable in order to simplify the joint estimation of  $\gamma$  and  $K$ , despite this standardization, we observed very large estimates of  $\gamma$  in most of the sectors. In Table 3, we found 5 of the sectors with  $\gamma < 5$  and the

**Table 3:** Estimates of LSTM Models for the Overall Series

Sectors	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}^U$	$\hat{\beta}^U$	$\hat{\gamma}$	$\hat{K}$
Agriculture	-0.01617 (0.0066)	-0.27463 (0.2732)	0.01693 (0.0067)	0.34681 (0.2775)	11811	-0.01553
Transportation	-0.00033 (0.0022)	0.09263 (0.2516)	0.00538 (0.0038)	-0.35384 (0.3805)	30706	0.00102
Finance	0.00073 (0.0005)	0.29964 (0.0540)	-59682.5 (0.0000)	1099274.1 (0.0000)	6.59829	0.05788
Food	0.00069 (0.0006)	0.32195 (0.0724)	0.02671 (0.0156)	-1.04499 (0.5662)	5.59728	0.01767
Construction	0.06174 (0.0390)	1.50487 (1.1489)	-0.06165 (0.0390)	-1.24419 (1.1565)	7.79795	-0.02434
Engineering & Technology	-0.00029 (0.0004)	0.07399 (0.0415)	0.07336 (0.0106)	-2.60672 (0.3878)	6791.84	0.02064
Footwear	-0.00075 (0.0014)	-0.03826 (0.0809)	0.00182 (0.0014)	-0.02295 (0.0860)	6573.63	-0.00843
Health	-0.00064 (0.0011)	0.02032 (0.1014)	0.00218 (0.0015)	-0.06033 (0.1367)	36.0036	-0.00060
Industrial Products	-0.00061 (0.0081)	0.11777 (0.1245)	0.00694 (0.0048)	-0.39575 (0.2395)	3.14922	0.00754
Building	-0.00001 (0.0001)	0.01698 (0.0163)	0.00643 (0.0031)	-0.21616 (0.1219)	3837.4081	0.01857
Conglomerate	-0.00115 (0.0013)	0.05310 (0.0952)	0.00179 (0.0014)	0.01634 (0.1115)	22.27126	-0.00382
Packaging	-0.00031 (0.0003)	0.06586 (0.0283)	0.02231 (0.0094)	-0.68879 (0.3237)	8735.088	0.02212
Petroleum	-0.00029 (0.0005)	0.19403 (0.0563)	0.04621 (0.0137)	-1.50791 (0.5065)	97.85877	0.02013
Printing	-0.00795 (0.0059)	-0.31560 (0.3511)	0.00866 (0.0060)	0.13878 (0.3826)	25.37859	-0.00695
Chemicals	-0.00595 (0.0026)	-0.27314 (0.1469)	0.00738 (0.0029)	0.26701 (0.1649)	5.86525	0.00445

Standard errors are reported in parenthesis under the estimates.

remaining 10 have very high values, which suggested that some sector equities experienced abrupt changes between bear and bull phases. These sectors with slow transition between bear and bull phases are Finance, Food, Construction, Industrial Products and Chemicals. Consistent with Harlow and Rao (1989) suggestion that investor targets do not just depend on parameters

computed with the distribution of market returns; there is no apparent common value for this parameter across different industries.

**Table 4:** Estimates of LSTM Models for the period before the Global Financial Crisis

Sectors	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}^U$	$\hat{\beta}^U$	$\hat{\gamma}$	$\hat{K}$
Agriculture	0.07620 (0.0000)	3.84274 (0.0000)	-0.07594 (0.0000)	-3.63910 (0.0000)	1.12535	-0.04361
Transportation	0.00087 (0.0031)	0.08384 (0.4095)	0.00728 (0.0053)	-0.55269 (0.5785)	90.9997	0.00078
Finance	0.00281 (0.0009)	0.71890 (0.1383)	-0.00352 (0.0031)	-0.15970 (0.2666)	199036	0.00483
Food	0.00492 (0.0026)	0.89530 (0.2312)	-0.00669 (0.0048)	-0.27696 (0.3473)	5.44033	0.00064
Construction	-0.00048 (0.0009)	0.07480 (0.1362)	0.01335 (0.0056)	-0.27288 (0.3761)	240.8016	0.00851
Engineering & Technology	0.00051 (0.0003)	0.05339 (0.0373)	0.00627 (0.0026)	-0.32555 (0.1441)	42.0485	0.01102
Footwear	0.00095 (0.0003)	-0.02572 (0.0418)	0.01281 (0.0082)	-0.49648 (0.3480)	317.25439	0.01759
Health	0.00181 (0.0007)	0.22437 (0.1132)	-0.00713 (0.0035)	0.20166 (0.2620)	47.52201	0.00669
Industrial Products	0.00081 (0.0007)	0.42974 (0.1017)	0.00943 (0.0054)	-0.78205 (0.3354)	32.37108	0.00973
Building	0.00008 (0.0001)	0.01019 (0.0098)	-0.00120 (0.0005)	0.05569 (0.0321)	62.89803	0.00945
Conglomerate	0.04737 (0.0611)	1.78521 (1.7901)	-0.04732 (0.0611)	-1.53001 (1.7883)	1.95266	-0.02696
Packaging	0.00024 (0.0001)	0.01070 (0.0129)	-0.00117 (0.0008)	0.05571 (0.00458)	28.86846	0.01048
Petroleum	0.00041 (0.0004)	0.23877 (0.0456)	2.12528 (0.6822)	-61.26900 (19.9732)	133.70077	0.03288
Printing	-0.01161 (0.0076)	-0.60131 (0.4854)	0.01445 (0.0078)	0.37268 (0.5117)	25.32094	-0.00732
Chemicals	-0.01035 (0.0052)	-0.48603 (0.2940)	0.01296 (0.0056)	0.47959 (0.3255)	4.38522	-0.00532

**NB:** Standard errors are reported in parenthesis under the estimates.

The up-market betas for the sectors are recorded in the  $\hat{\beta}^U$  column, and 11 of these are significant at 5% level. We can see that the LSTM model therefore provide widespread support for time varying betas, with variations linked to movements in the business cycle. Eight of the eleven statistically significant up market betas are negative; this is in agreement with the literature that risk in up market is lower than that in the down-markets.



**Table 5:** Estimates of LSTM Models for the period after the Global Financial Crisis

Sectors	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}^U$	$\hat{\beta}^U$	$\hat{\gamma}$	$\hat{K}$
Agriculture	-0.02684 (0.0089)	-0.76943 (0.3669)	0.02768 (0.0089)	0.72447 (0.3745)	25.58852	-0.01500
Transportation	-0.00067 (0.0027)	0.15000 (0.2832)	-0.00521 (0.0072)	0.19578 (0.5435)	200978	0.00408
Finance	-0.00133 (0.0007)	0.04335 (0.0000)	0.01511 (0.0094)	-0.53924 (0.3887)	1168.939	0.01542
Food	-0.00013 (0.0008)	0.15647 (0.0840)	0.03739 (0.0148)	-1.33213 (0.5393)	8.54906	0.01754
Construction	0.10070 (0.0454)	2.17242 (1.3330)	-0.10198 (0.0454)	-2.01154 (1.3378)	15.89829	-0.12631
Engineering & Technology	-0.00160 (0.0009)	0.05963 (0.0870)	0.08458 (0.0183)	-2.88614 (0.6501)	62.93592	0.02026
Footwear	-0.00100 (0.0013)	-0.02068 (0.0740)	0.00201 (0.0013)	-0.07444 (0.0803)	687.96581	-0.00813
Health	-0.01604 (0.10155)	-0.48717 (0.5253)	0.01541 (0.0155)	0.51464 (0.5286)	29.13244	-0.02288
Industrial Products	0.00409 (0.0053)	0.15798 (0.2379)	-0.00555 (0.0053)	-0.22344 (0.2477)	21.57934	-0.01217
Building	-0.00006 (0.0003)	0.02947 (0.0358)	0.01035 (0.0058)	-0.33675 (0.2217)	3777.230	0.01857
Conglomerate	-0.00309 (0.0029)	-0.11442 (0.1556)	0.00403 (0.0031)	0.00576 (0.1689)	10.01772	-0.00518
Packaging	-0.00097 (0.0006)	0.10910 (0.0622)	0.03559 (0.0172)	-1.03764 (0.5716)	9738.0865	0.02212
Petroleum	-0.00159 (0.0011)	0.11192 (0.1082)	0.09591 (0.0231)	-2.92724 (0.8223)	6076.0114	0.02065
Printing	0.11339 (0.0692)	3.14433 (2.0395)	-0.11640 (0.0692)	-3.11567 (2.0436)	15.79602	-0.02638
Chemicals	-0.00147 (0.0005)	-0.05384 (0.0482)	0.00763 (0.0037)	-0.21275 (0.1717)	9.30564	0.01033

**Note:** Standard errors are reported in parenthesis under the estimates

We therefore divided the data into two sub-series with February 2008 as the break point. We estimated LSTM model for each of the sub-series (before and after financial crisis). The results are presented in Tables 4 and 5.

In Table 4, the transition between bull and bear period is abrupt in most of the portfolios. The speed is highest for financial sector and lowest for conglomerates sector.

After the financial crisis (starting from March, 2008) when the stock market started experiencing the global shock, we have similar results of LSTM models in Table 5. We found that Packaging, Transportation, Petroleum and Building experienced abrupt change between market phases. This implies that these portfolios (industries) recovered fast after the crisis.

## 5.0 Policy Implications and Conclusion

Financial analysts believe that market and portfolio betas are influenced by the alternating forces of bull and bear markets. Most of the studies have applied simple threshold model which classifies market in these two phases. This work is the first of its kind classifying Nigerian stock markets into bull and bear phases using the logistic smooth threshold market (LSTM) model. Using the All share Index (ASI) and selected 15 portfolios (sectors/industries on Nigerian Stock Exchange), we tested for up (bull) and down (bear) markets differentials using the logistic smooth transition nonlinearity test similar to that proposed in Luukkonen, et al. (1988) and applied in Teräsvirta (1994).

The results obtained showed strong evidence of betas varying between bull and bear phases. The estimates of LSTM model indicated that the transition is fast (abrupt) for most of the portfolios and this is in support of the homogenous beliefs among the investors as a result of news/information symmetry. Also, the up-market and down-market betas are significantly different in most of the portfolios.

Our results are consistent with Pagan and Sossounov (2003) and Cunado *et al.* (2008) who found stocks spending more time in bull-market than bear-market states. This work has offered an alternative way of studying Nigerian stock market asymmetries.

Findings of this research have policy implications. Within the period under study, the CPM identified the Petroleum, Finance and Food to be of higher risk as compared to aggregate market risk of all sectors. We also found that for most industries, the beta estimate obtained before financial crisis is different from that obtained after the financial crisis.

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