On Fractionally Integrated Logistic Smooth Transitions in Time Series

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Long memory and nonlinearity are two key features of some macroeconomic time series which are characterized by persistent shocks that seem to rise faster during recession than it falls during expansion. A variant of nonlinear time series model together with long memory are used to examine these features in inflation series for three economies. The results which compares favourably with that of van Dijk et al. (2002) elicit some interesting attributes of inflation in the developed and developing economies.

Keywords: Fractional integration, Long memory, Smooth transition autoregression, Inflation rates, Time series.

JEL Classification: C22, C51, C87.

1. Introduction

The era of nonlinear modelling has come to complement linear modelling in financial or econometric time series. This is due to the fact that many real world problems do not satisfy the assumptions of linearity and/or stationarity. The classical theory of stationarity and linearity may not apply to some economic, finance and macroeconomic series because they consider series at its level, $I(0)$; first order integrated series, $I(1)$ as well as higher order integrated series (Box and Jenkins, 1976). Hassler and Wolters (1995) considered a case of long memory, $I(0<d<0.5)$ for inflation data from five industrialized countries and found that the series are all within the long memory range.

The nonlinearity property of economic series can also be justified by the existence of asymmetry in inflation's dynamics (Mourelle *et* al., 2011). In order to consider these possible nonlinearities, it is necessary to have econometric models that are able to generate different dynamics according to the business cycle phase. (see Granger and Teräsvirta (1993); Teräsvirta (1994)). van Dijk *et* al. (2002) present the modelling cycle for specification of smooth transition autoregressive (STAR) model which include estimation of differencing parameter, testing for nonlinearity, parameter estimation and model adequacy tests, in the case where the transition function is the logistic function and applied this on US monthly unemployment rate. Smallwood (2005) and Boutahar *et* al. (2008) extend these results to the fractionally integrated STAR (FISTAR) model with an exponential transition function. The model was applied to measure the purchasing power

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by considering the real exchange rate data for twenty countries. This model is still new and has to be tried beyond its applicability to exchange rates.

This paper therefore seeks to examine the dynamics and application of fractionally integrated logistic STAR (FILSTAR) model on inflation rates with a view to obtaining better parameter estimates and reliable forecasts. The remaining sections of the paper are then organized as follows: Section 2 gives the general review of the FISTAR model and the linearity tests. Section 3 discusses the estimation of the model; Section 4 presents the results of the analysis and Section 5 gives the conclusion.

2. The FISTAR Model Specification

A Fractionally Integrated (FI) time series process $\{X_t\}, t=1,...,T$ is considered as

$$(1-B)^d X_t = y_t \tag{1}$$

Where B is the backward shift operator, d is the non-integer fractional differencing parameter and y_t is a covariance-stationary process. For fractionally integrated process in (1), the integration parameter d assumes values within the stationary and invertible range $-0.5 < d < 0.5$ (Sowell, 1992a; Mayoral, 2007).For $0 < d < 0.5$, X_t is a stationary long memory process in the sense that autocorrelations are not absolutely summable but rather at a much slower hyperbolic rate. It exhibits nonstationary process if $0.5 \le d < 1$.

Applying the Maclaurin's series expansion around $B=0$, the fractional difference operator is expanded as,

$$(1-B)^d = 1 - dB - \frac{d(1-d)B^2}{2!} - ... = \sum_{j=0}^{\infty} \frac{\Gamma(-d+j)}{\Gamma(-d)\Gamma(j+1)} B^j \tag{2}$$

where the Euler gamma function,

$$\Gamma(z) = \int_0^{\infty} s^{z-1} e^{-s} ds = (z-1)! \quad z > 0 \tag{3}$$

Based on (2) and (1), the fractionally integrated STAR (FISTAR) model of order p is expressed as,

$$\begin{aligned} &\sum_{j=0}^{\infty} \frac{\Gamma(-d+j)}{\Gamma(-d)\Gamma(j+1)} B^j X_t = y_t \\ &y_t = \phi_1' \tilde{y}_t^{(p)} \left(1 - F(s_t;\gamma,c)\right) + \phi_2' \tilde{y}_t^{(p)} F(s_t;\gamma,c) + \varepsilon_t \end{aligned} \tag{4}$$

where $t=1,2,...,T$, $\tilde{y}_t^{(p)} = (1, y_{t-1},..., y_{t-p})'$, $\phi_i = (\phi_{i0}, \phi_{i1},...,\phi_{ip})'$ and i=*1,2*. The ε_t is assumed to be a difference sequence distributed with $E(\varepsilon_t | \Omega_{t-1}) = 0$ and $E(\varepsilon_t^2 | \Omega_{t-1}) = \sigma^2$ with

$\Omega_{t-1} = y_{t-1}, y_{t-2}, ..., y_{1-(p-1)}, y_{1-p}$ representing the past history of the time series. Following Teräsvirta (1994), the transition variable s_t is assumed to be a lagged endogenous variable, that is, $s_t = y_{t-l}$ for certain integer $0 < l \leq p$. At point l, nonlinearity is sharper. For the case where 1 or 2 autoregressive parameters determine the linear part of the STAR model, the inequality $l > p$ holds. In the general FISTAR model in (4) above, the transition function $F(s_t;\gamma,c)$ is assumed to be either of logistic or exponential form (Teräsvirta, 1994) as given below:

$$F(s_t;\gamma,c) = \frac{1}{1+\exp(-\gamma(s_t - c))}, \quad \gamma > 0 \tag{5}$$

$$F(s_t;\gamma,c) = 1-\exp\left(-\gamma(s_t - c)^2\right), \quad \gamma > 0 \tag{6}$$

In that case, using either (5) or (6) in the FISTAR model in (4) leads to fractionally integrated logistic STAR (FILSTAR) and fractionally integrated exponential STAR (FIESTAR) models respectively. The γ is the slope parameter and c, the intercept in the transition function. In the FILSTAR and FIESTAR models mentioned above, it is clear that the models reduces to linear autoregressive fractionally integrated (ARFI) of order p when the transition function, $F(s_t;\gamma,c)=0$ or 1, that is shifting between two extreme linear regimes after staying in nonlinear region for some time. The fractional parameter *d*, the autoregressive parameters, ϕ_i and nonlinear parameters, γ and c make the FISTAR model potentially useful for capturing both long memory and nonlinear smooth transition features of the time series, X_t (Boutahar *et al*., 2008).

STAR modelling approach of Teräsvirta (1994) has been modified to capture our specification procedure for FISTAR, as it is proposed by van Dijk et al. (2002):

1. Specify a linear ARFI (*p*) model by selecting the autoregressive order *p* by means of Akaike and Schwarz information criteria (Akaike, 1974 and Schwarz, 1978).
2. Test the null hypothesis of linearity against the alternative of a FISTAR model.
3. Specify the model STAR model by choosing between the two competing transition functions.
4. Estimate the parameters in the specified FISTAR model.
5. Evaluate the estimated model using misspecification tests (no residual autocorrelation, serial correlation, normality test, ARCH test and others).

Teräsvirta (1994) follows the approach of Luukkonen, Saikkonen and Teräsvirta (1988) in replacing the transition function $F(s_t;\gamma,c)$ with a suitable Taylor series approximation about $\gamma = 0$ and test linearity by means of a Lagrange multiplier (LM) statistic. Luukkonen *et al*. (1988) then consider testing the null hypothesis $H_0 : \gamma = 0$ of linearity against the alternative of logistic STAR (LSTAR) nonlinearity by using the LSTAR function. The third order Taylor's series approximation of the logistic model is then given as auxiliary regression model,

$$y_t = \phi' \tilde{y}_t^{(p)} + \beta_1' \tilde{y}_t^{(p)} s_t + \beta_2' \tilde{y}_t^{(p)} s_t^2 + \beta_3' \tilde{y}_t^{(p)} s_t^3 + \tilde{\varepsilon}_t \tag{7}$$

where $\beta_i = (\beta_{i1},...,\beta_{ip})'$, $i = 1,2,3$ are functions of the parameters ϕ_1, ϕ_2, γ and c. The null hypothesis then becomes $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$, which implies the selection of linear model. The approach of Teräsvirta (1994) is to specify the model based on the nested hypotheses:

$$H_{O1} : \beta_3 = 0$$
$$H_{O2} : \beta_2 = 0 | \beta_3 = 0 \; H_{O3} : \beta_1 = 0 | \beta_2 = \beta_3 = 0 \quad (8)$$

which is supported by Escribano and Jordá (2001). This sequence of hypotheses implies that rejection of H_{O1} suggests acceptance of LSTAR model. Also, rejection of H_{O2} is an acceptance of exponential STAR (ESTAR) model. Lastly, rejection of H_{O3} implies the selection of LSTAR model.

Analytically, the test procedures follow:

1. Regressing y_t on $\{1, y_{t-j}; j = 1,2,..., p\}$ to form $\hat{\varepsilon}_t$, $(t = 1,2,...,T)$ and computing the residual sum of squares $SSR_0 = \sum_{t=1}^{T} \hat{\varepsilon}_t^2$;
2. regressing $\hat{\varepsilon}_t$ on $\{1, y_{t-i}, y_{t-i}s_t^k,; i = 1,2,..., p; k = 1,2,3\}$ to form the residuals $\tilde{\varepsilon}_t \; (t = 1,2,...,T)$ and $SSR_1 = \sum_{t=1}^{T} \tilde{\varepsilon}_t^2$ and
3. computing the test statistic $F = \dfrac{(SSR_0 - SSR_1)/3(p+1)}{SSR_0/(T-4(p+1))}$ and $F \approx F_{3(p+1),\; T-4(p+1)}$

3. Estimation of FISTAR Parameters

The estimation of FISTAR model starts by estimating the fractional difference parameter in the series. This is achieved using Hurst (1951) by computing the Hurst coefficient. He used the non-parametric approach by employing a rescaled statistic (R/S) defined as:

$$R/S = 1/S_T(q)\left(\sup_{1\le m\le T}\sum_{j=1}^{m}\left(X_j - \overline{X}\right) - \inf_{1\le m\le T}\sum_{j=1}^{m}\left(X_j - \overline{X}\right)\right) \quad (9)$$

where S_T is the MLE estimate of standard deviation from time series, X_j. Then, $S_T(q) = S_T + 2\sum_{j=1}^{q} w_j(q)\hat{\gamma}_j$ and $w_j(q) = 1 - j/(q+1)$ such that $q < T$ (Lo, 1991).The Hurst coefficient, H is then estimated by,

$$\hat{H} = \frac{1}{\log(T)}\log(R/S). \quad (10)$$

The fractional differencing parameter, d is then obtained as,

$$\hat{d} = \hat{H} - 0.5. \quad (11)$$

The approximate values of y_t can be obtained in the time domain as in Sowell (1992a, b). The time domain approach follows the Binomial Theorem representation of $(1-B)^d$. This implies that y_t is approximated by using $\hat{d}$ estimated by the Hurst estimation approach and a truncated fractionally differenced series is given as,

$$\tilde{y}_t = \sum_{j=t}^{\infty} \frac{\Gamma\left(-\hat{d}+j\right)}{\Gamma\left(-\hat{d}\right)\Gamma\left(j+1\right)} X_{t-j} \tag{12}$$

$$= \sum_{k=0}^{\infty} \frac{\Gamma\left(-\hat{d}+t+k\right)}{\Gamma\left(-\hat{d}\right)\Gamma\left(t+k+1\right)} X_k \tag{13}$$

From (12), it is clear to set $X_{t-j}=0$ for $t-j$ outside of the sample, *T*.

The second transformation approach uses the frequency domain approach of Geweke and Porter-Hudak (1983). Here, the Fourier transform of the observed series, X_t is pre-multiplied by the Fourier transform of the fractional differencing operator based on $\hat{d}$, and then compute the inverse Fourier transform. The final series obtain follows an autoregressive moving average (ARMA (*p*,*q*)) process.

According to van Dijk et. al. (2002), after the estimation of the fractional difference parameter, all the remaining parameters in the STAR model are estimated together. Beran (1995) suggests approximate maximum likelihood (AML) estimator for invertible and possible nonstationary autoregressive fractionally integrated moving average (ARFIMA) model which allows for regime switching autoregressive dynamics. This estimator then minimizes the sum of squared residual of the STAR model as,

$$S\left(\phi_1,\phi_2,\gamma,c\right) = \sum_{i=1}^{T} \varepsilon_t^2\left(\phi_1,\phi_2,\gamma,c\right). \tag{14}$$

We now consider the choice of appropriate starting value parameters and the estimation of the smoothness parameter in the transition function. The estimation procedure can be simplified by concentrating the sum of squares function since the parameters γ and c in the transition function imply STAR model of parameters ϕ_1 and ϕ_2 and this makes the FISTAR model linear in the remaining parameters (Leybourne *et* al. (1998),van Dijk et al. (2002)). Then, estimates of $\phi = \left(\phi_1', \phi_2'\right)'$ can be obtained by ordinary least squares (OLS) as

$$\hat{\phi}\left(\gamma,c\right) = \frac{\sum_{t=1}^{T} \tilde{y}_t^{(p)}\left(\gamma,c\right) y_t}{\sum_{t=1}^{T} \tilde{y}_t^{(p)}\left(\gamma,c\right) \tilde{y}_t^{(p)}\left(\gamma,c\right)'} \tag{15}$$

where $\tilde{y}_t^{(p)}\left(\gamma,c\right) = \left[\tilde{y}_t'^{(p)}\left(1-F(s_t;\gamma,c\right), \tilde{y}_t'^{(p)}F(s_t;\gamma,c)\right]'$ and the notation $\hat{\phi}\left(\gamma,c\right)$ is used to indicate that the estimate of ϕ is conditional upon γ and c. Thus, the sum of squares function $Q_N(\varphi)$ can be concentrated with respect to ϕ_1 and ϕ_2 as,

$$Q_N(\gamma,c)=\sum_{t=1}^{T}\left[y_t-\hat{\phi}(\gamma,c)'\tilde{y}_t^{(p)}(\gamma,c)\right]^2 \tag{16}$$

and $Q_N(\gamma,c)$ will be minimized with respect to parameters γ and c only. The estimate of γ is very difficult to obtain when it is large because its large value makes the STAR model to be similar to threshold autoregressive (TAR) model as the transition function, $F(s_t;\gamma,c)$ comes close to a step function and this function is then standardized. To obtain an accurate estimate, there should be many observations, s_t in the neighbourhood of c and this implies small deviation. Sensible starting value for the nonlinear optimization of the STAR model can easily be obtained by considering a two dimensional grid search over γ and c. Then for any set of the two values $\left(A_j^c, A_k^\gamma\right)$, the parameter vectors (ϕ_1,ϕ_2) are then estimated through ordinary least squares (OLS). The outcome of this is a set of estimates, $\left(\hat{\phi}_1^0,\hat{\phi}_2^0,\hat{\gamma}^0,\hat{c}^0\right)$. Practically, most estimation software for STAR modelling are designed to follow the specification,

$$y_t=\phi_1'\tilde{y}_t^{(p)}+\left(\phi_2-\phi_1\right)'\tilde{y}_t^{(p)}F\left(s_t;\gamma,c\right)+\varepsilon_t \tag{17}$$

where the nonlinear part is on one side of the model.

4. Data Analysis and Results

The monthly macroeconomic time series data on inflation are sourced variously from Federal Reserve Bank of St. Louis (US Inflation data), National Bureau of Statistics (Nigerian Inflation data) and Office of National Statistics (UK Inflation data). These series range from January 1991 to December 2009 (T = 228). These data are large enough to adjust for the lag operations performed during model specification and estimation. Preliminary analyses have been performed using EViews 5 software from Quantitative Micro Software, LLC. Smooth Transition Regression (STR) analysis is performed using the R-STAR contributed package available through R Development Core Team (2009) for the analysis of nonlinear time series (Balcilar, 2008).

Table 1: Descriptive Statistics on Time Series

Statistics	Nigeria Inflation Series	US Inflation Series	UK Inflation Series
Minimum	0.90	-2.10	0.50
Maximum	78.50	5.65	8.50
Mean	22.020	2.650	2.351
Std. Dev.	19.856	1.179	1.574
Skewness	1.362	-0.867	2.252
Kurtosis	3.573	5.855	8.063
Jarque-Bera	73.596	105.957	436.189
Probability	0.0000	0.0000	0.0000

Table 1 shows the significance of the Jarque-Bera test of normality at 5% level for inflation series in Nigeria (NIIR), US (USIR) and the UK (UKIR) which implies that inflation rates are not normally distributed. Nigerian inflation rates rose as high as 78.50 between 1995 and 1996.The minimum inflation rate was experienced in 2000. US and UK displayed fair level of stable inflation between 1991and 2008. The time plots clearly display asymmetric behaviours and high persistence of inflation over the years and this is in accordance to van Dijk *et* al. (2002).

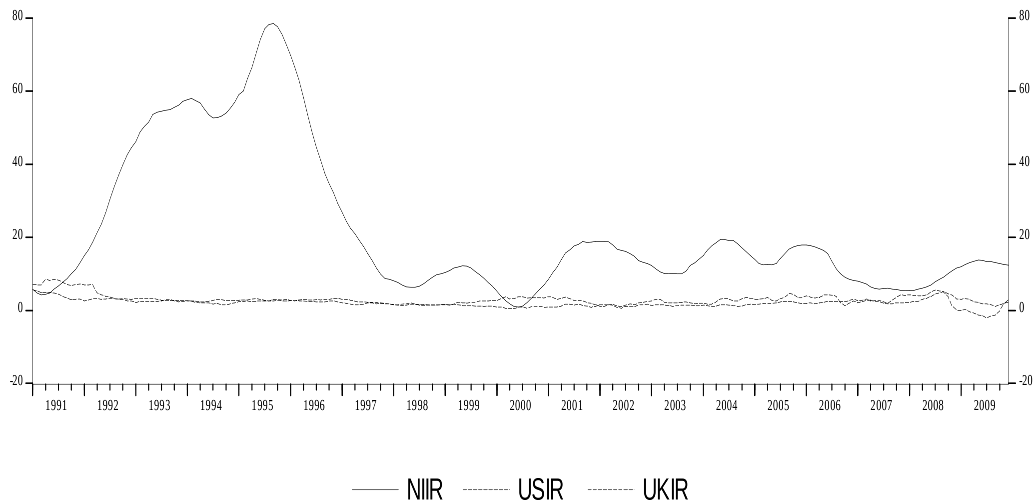

Fig. 1: Time Plots of Inflation Rates

Probing further into the dynamics of stationarity based on augmented Dickey Fuller (ADF) unit root tests which has the null hypothesis of unit root. The results of the ADF test indicate rejection of this null hypothesis at 1, 5 and 10% for Nigeria and US inflation series. UK inflation is stationary at 5 and 10% level. The ADF test is confirmed using the KPSS test of long memory as reported jointly with ADF test in Table 1. This test has been applied in Lee and Schmidt (1996) to test for stationarity in long memory range. It has the null hypothesis of series stationarity against long memory. Subjection of our inflation series to KPSS shows rejection of null hypothesis. Therefore, long memory is confirmed in the inflation series.

Table 2: Stationarity Tests on Inflation Time Series

	Nigerian Inflation Series(NIIR)			US Inflation Series (USIR)			UK Inflation Series (UKIR)		
Test	ADF		KPSS	ADF		KPSS	ADF		KPSS
Statistic	*-3.522*		*0.752*	*-4.589*		*0.737*	-3.055		1.212
1%	-3.459		0.739	-3.459		0.739	-3.459		0.739
5%	-2.874	(0.0083)	0.463	-2.874	(0.0002)	0.463	-2.874	(0.0083)	0.463
10%	-2.574		0.347	-2.574		0.347	-2.574		0.347

Since fractional differencing is possible in the inflation series based on stationarity tests, Table 3then shows the estimates of the fractional difference parameter computed after Hurst (1951). The fractional difference estimates are actually in long memory range. The estimates of 0.3289 reported for Nigeria is another indication of increase in the inflation rate as compared with US and UK inflation rates.

Table 3: Estimation of Fractional Difference Parameters

	Non-Parametric Approach		
	Nigerian Inflation Series (NIIR)	US Inflation Series (USIR)	UK Inflation Series (UKIR)
$\hat{d}$ *R/S*	0.3289 (90.0325)	0.1542 (34.8870)	0.2819 (69.7733)

The series are then fractionally differenced based on the estimates in Table 3 to have "pure" stationary series. The transformed series, as subjected to stationarity tests in Table 4 give acceptance of null hypothesis of stationarity of KPSS test as against the alternative hypothesis. Fractional differencing actually removed long memory effects in the series.

Table 4: Stationarity Tests on Fractionally Differenced Time Series

	Nigerian Inflation Series (NIIR)			US Inflation Series (USIR)			UK Inflation Series (UKIR)		
Test	ADF		KPSS	ADF		KPSS	ADF		KPSS
Statistic	*-3.122*		*0.416*	*-3.186*		*0.1639*	-6.046		0.2543
1%	-3.459	(0.0264)	0.739	-3.459	(0.0222)	0.739	-3.459	(0.0000)	0.739
5%	-2.874		0.463	-2.874		0.463	-2.874		0.463
10%	-2.574		0.347	-2.574		0.347	-2.574		0.347

Modelling cycle of FISTAR model continues by fitting linear AR models to the inflation series. Optimal models were obtained based on minimum values of AIC and SIC. So, AR (2), AR (4) and AR (2) models are optimal models for Nigeria, US and UK inflation series respectively. Table 5 shows the result of the first stage in nonlinear STAR testing. Nonlinearity is found to be sharper at different lags, $l = 4$, $l = 3$ and $l = 1$ for Nigerian, US and UK inflation series. These are determined as least significant points for $0 < l \leq p$ or $l > p$. Note that this is determined based on $l > p$, that is certain point outside the model lags. Nonlinear smooth transitions are tested in these sharper points. The test results as given in Table 6 shows at least the significance of one of the β_i based on the auxiliary regression in (7) which is an indication that the three inflation series exhibit nonlinear smooth transition autoregressive behaviour.

Table 5: Determination of the Transition Variable, $s_t = y_{t-l}$

	Nigeria Inflation Series					US Inflation Series				UK Inflation Series			
Dela	1	2	3	4	5	1	2	3	4	1	2	3	4
Prob	0.013	0.010	0.007	***0.005***	0.007	0.000	0.000	***0.000***	0.000	***0.224***	0.318	0.327	0.418

Based on the nested hypothesis in (8), LSTAR models are specified for the three inflation rates unlike ESTAR model specified for exchange rates in (Boutahar, 2008). The specification of LSTAR model for inflation series support the fact that inflation series are assymetric.

Table 6: STAR Nonlinearity Test and Model Specification

	Nigeria Inflation Series			US Inflation Series			UK Inflation Series		
β_i	1	2	3	1	2	3	1	2	3
Prob..	0.37617	***0.00094216***	0.43241	***0.031438***	***8.34E-07***	***0.00019***	0.60882	0.69793	***0.03607***
Model	LSTAR			LSTAR			LSTAR		

Estimation results are presented in Tables 7-9. Note subset of the insignificant nonlinear parameters cannot be taken in the RSTAR contributed package for R software. So, the STAR and FISTAR models presented are optimal.

Table 7: Estimated LSTAR Model for Nigerian Inflation Rates

	Model	AR		ARFI		LSTAR		FILSTAR	
	Estimator	Estimates	Prob.	Estimates	Prob.	Estimates	Prob.	Estimates	Prob.
Linear part	$\hat{\phi}_{10}$	21.65910	0.00000	1.64978	0.0000	0.23993	0.0001	0.03211	0.4513
	$\hat{\phi}_{11}$	1.93548	0.00000	-0.48204	0.0001	1.98177	0.0000	1.58115	0.0000
	$\hat{\phi}_{12}$	-0.94128	0.00005	-0.18114	0.0030	-1.00173	0.0000	-0.32336	0.0538
	$\hat{\phi}_{13}$							-0.26088	0.0035
Nonlinear part	$\hat{\phi}_{20}$					2.79471	0.0398	1.65748	0.2963
	$\hat{\phi}_{21}$					-0.03051	0.0309	-0.64939	0.0815
	$\hat{\phi}_{22}$							0.17342	0.7273
	$\hat{\phi}_{23}$							0.95191	0.0324
	$\hat{\phi}_{24}$							-0.67563	0.0014
	$\hat{\gamma}$					2.92593	0.1887	10.98377	0.3251
	$\hat{c}$					43.88512	0.0000	15.10189	0.0000
	$\hat{l}$					4		4	
Diagnostic tests	*AIC*	332.0028		310.4322		300.2108		299.1587	
	SIC	342.2908		321.5621		324.0610		336.6868	
	R^2	0.9994		0.9959		0.9995		0.9963	
	ARCH-LM	0.5657	0.0452	1.8344	0.0177	2.08314	0.0150	3.43958	0.0650

In this Table, the point estimates of slope parameters $\hat{\gamma} = 2.92592$ for LSTAR and $\hat{\gamma} = 10.98377$ for FISTAR models indicate that the transition between the two regimes of STAR model is slow and fast. Fractional integration, in fact led to improved fit as indicated in the estimates of R^2.This implies that ARFI and FISTAR models are preferred to AR and STAR models respectively but the introduction of FI does not lead to improve fit in the FISTAR model (van Dijk et al., 2002).

Table8: Estimated LSTAR Model for US Inflation Rates

	Model	AR		ARFI		LSTAR		FILSTAR	
	Estimator	Estimates	Prob.	Estimates	Prob.	Estimates	Prob.	Estimates	Prob.
Linear part	$\hat{\phi}_{10}$	2.72725	0.0000	1.24780	0.0000	0.16843	0.1370	0.01088	0.6068
	$\hat{\phi}_{11}$	1.43073	0.0000	-0.54730	0.0000	1.28605	0.0000	1.16477	0.0000
	$\hat{\phi}_{12}$	-0.73275	0.0000	0.17335	0.0113	-0.52379	0.0000	-0.49817	0.0000
	$\hat{\phi}_{13}$	0.36304	0.0024			0.17673	0.0239	0.24095	0.0084
	$\hat{\phi}_{14}$	-0.14600	0.0397						
Nonlinear part	$\hat{\phi}_{20}$					5.36344	0.0818	-1.67135	0.3084
	$\hat{\phi}_{21}$					1.13672	0.0443	0.86859	0.3403
	$\hat{\phi}_{22}$					-2.24124	0.0123		
	$\hat{\gamma}$					7.67672	0.5270	6.05677	0.4182
	$\hat{c}$					4.90000	0.0000	1.64402	0.0000
	$\hat{l}$					3		3	
Diagnostic tests	*AIC*	206.8093		200.4433		179.6015		186.8999	
	SIC	223.9560		213.4328		210.3063		214.2287	
	R^2	0.8953		0.8223		0.9067		0.8383	
	ARCH-LM	1.9750	0.0118	1.5736	0.0196	2.1640	0.0142	4.3252	0.0377

Table 8 also shows the estimated slope parameters of $\hat{\gamma} = 7.67672$ for LSTAR and $\hat{\gamma} = 6.05677$ for FISTAR models indicating that the transition between the two regimes of STAR model is slow. The values of the slope parameters are closed to each other because of small value of the difference parameter, $\hat{d} = 0.1542$. Also, FI led to improved fit but STAR model is preferred to FISTAR model.

From Table 9, value of the slope parameter dropped from $\hat{\gamma} = 33.7800$ to $\hat{\gamma} = 6.05677$ indicating fast to slow smooth transitioning from one regime to the other.FI also improved the model fit from AR and STAR to ARFI and FISTAR models respectively.

Conclusion

This paper has considered a model proposed in Van Dijk*et* al. (2002) to model macro econometric time series that is asymmetric. The model is found to be able to describe both long memory and nonlinearity through fractional integration and smooth transition modelling. Inflation dynamics display high persistence which is an evidence of long memory. Stationary time series models can be improved by fractionally integrating the series. Also, time series model can be improved upon by considering and modelling nonlinearity in the series. We would have expected FILSTAR model to be the better one out of the four models for each of the inflation series but this is not the case for the three series. This indicates the serious competition arising between fractional integration and nonlinearity of series.

Table9: Estimated LSTAR Model for UK Inflation Rates

	Model	AR		ARFI		LSTAR		FILSTAR	
	Estimator	Estimates	Prob.	Estimates	Prob.	Estimates	Prob.	Estimates	Prob.
	$\hat{\phi}_{10}$	3.285163	0.0065			0.133100	0.0164	0.01088	0.6068
Linear part	$\hat{\phi}_{11}$	1.12114	0.0000	0.84805	0.0000	1.06600	0.0000	1.16477	0.0000
	$\hat{\phi}_{12}$	-0.13552	0.0443			-0.13540	0.0354	-0.49817	0.0000
	$\hat{\phi}_{13}$							0.24095	0.0084
	$\hat{\phi}_{20}$					1.59100	0.5418	-1.67135	0.3084
	$\hat{\phi}_{21}$					-0.184300	0.5709	0.86859	0.3403
Nonlinear part	$\hat{\gamma}$					33.7800	1.0000	6.05677	0.4182
	$\hat{c}$					6.02500	1.0000	1.64402	0.0000
	$\hat{l}$					1		1	
	AIC	137.9201		197.5456		129.4520		186.8999	
	SIC	148.2081		201.0211		153.3958		214.2287	
Diagnostic tests	R^2	0.9566		0.8335		0.9574		0.8383	
	ARCH-LM	0.0003	0.0985	0.0059	0.0939	0.05279	0.0818	1.70649	0.0193

Future research work should consider forecasts performance of these models. Fractional integration can also be combined with some other nonlinear time series models in order to confirm the inability FI-nonlinear model to give best fit. With this, the dominant feature between long memory and nonlinearity may be assessed.

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